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Abstract. Many industries operate technologies in which multiple outputs (coproducts) are jointly produced. In some settings ("vertical") the coproducts differ along a performance dimension and are substitutable. In other settings ("horizontal") the coproducts differ in their applications and are not substitutable. In both cases, three important attributes of a coproduct technology are its processing cost, overall yield, and coproduct split, i.e., the proportion of each output produced. For both vertical and horizontal settings with deterministic market sizes, we characterize the optimal pricing and production decisions of a monopoly firm with two technologies. We establish the necessary and sufficient conditions for dual activation (i.e., using both technologies) to be optimal. Dual activation is driven by differences in marginal costs across the two technologies. There is an additional motive for dual activation in the horizontal setting: the desire to generate a product mix that better resembles the market mix. Building on the optimal production analysis, we characterize the optimal adoption-and-usage strategy of a firm with one incumbent technology considering a new technology. We establish the conditions for which a new technology will displace the incumbent or be used with the incumbent, and highlight some important adoption and usage differences between vertical and horizontal settings. Results are extended to the setting with uncertain market size(s).

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1. Introduction

Multiple devices are built on a single wafer during semiconductor fabrication. The devices are then tested, with the good devices oftentimes being sorted into different quality grades ("bins") based on their evaluated performance on some key metric, e.g., speed for microprocessors or luminescence for light emitting diodes (LEDs). Industrial diamonds are produced by applying pressure and temperature to a capsule containing graphite, some of which is converted to diamond. The diamonds are tested and the good ones are sorted into different quality (strength) grades. These are both examples of coproduct technologies whereby production jointly produces multiple outputs. In semiconductors and diamonds, coproducts arise because production generates a spectrum of outputs that differ in quality. Coproducts are a common phenomenon in a range of other industries. Chemical reactions often produce coproducts. For example, the chloralkali process eletrochemically separates brine (a salt and water mixture) into chlorine and caustic soda. The cumene process oxidizes cumene to produce cumene hydroperoxide, which is then decomposed into phenol and acetone. In other settings, nature creates a commingled material that can be separated into constituent parts. For example, distillation separates crude oil into various hydrocarbon coproducts, and milling separates corn into starch (which can be converted to ethanol) and other coproducts.

The production economics of a coproduct technology are driven by its cost per base unit processed (e.g., cost per wafer start in semiconductors), its overall yield per base unit (e.g., number of good devices generated per wafer start), and its coproduct split, that is, the percentage of the overall yield accounted for by each coproduct (e.g., the percentage of good devices testing out to each grade; this is sometimes referred to as the "binsplit" in the semiconductor industry). These drivers—cost, yield, and split—are often closely guarded secrets, especially in semiconductors and industrial diamonds, but data for the yield and split have been reported for some technologies. For example, the overall yield of chlorine and sodium hydroxide in the chloralkali process is 1.22 tonnes for every tonne of salt processed (recall the salt is mixed with water) and the split is 47% chlorine and 53% sodium hydroxide (IPPC 2001). The cumene process's overall yield of acetone and phenol is 1.20 tonnes per

tonne of cumene processed (remember cumene is combined with oxygen) and the split is typically 38% acetone and 62% phenol (McKetta 1990).

To improve their production economics, firms devote significant effort to developing new technologies that improve the cost, yield, and/or split. For example, Bayer was a finalist for the 2013 German Industry Innovation Prize for a new chloralkali membrane-separation technology that purportedly reduces the processing cost by 15% because of a 30% reduction in energy consumption (Katz 2014). CB&I and Versalis are jointly marketing a new cumene process technology "which improves overall yield" by 3% but does not change the split (CBI 2014, pp. 1–2). Meanwhile, Shell has developed a new cumene process technology that reportedly has a higher cost but a lower acetone split because of the production of an additional coproduct (Nexant 2007). In semiconductor manufacturing, the technology has periodically moved to larger wafer sizes, e.g., from 200 mm to 300 mm wafers, which results in a higher processing cost per wafer but also a higher overall yield per wafer. To date, the overall yield has increased more than the cost, resulting in a lower cost per device.[1] Applied Materials undertook an LED process development project to increase the split of higher-quality devices (Cooke 2010). The Winter process (adopted by GE and DeBeers) increased the overall yield and the split of higher grade diamonds but had a higher cost per capsule processed (Sung et al. 2006).

Upon adopting a new technology, i.e., one that differs in the cost, yield, and/or split from its incumbent technology, a firm faces the question of how best to use the two technologies. Should it produce using both technologies or should it use one technology exclusively (presumably the new technology or why else adopt it)? In determining whether or not to adopt a new technology, the firm must evaluate how the new technology will be used (exclusively or in conjunction with the incumbent) and also consider the adoption cost; that is, the cost of installing the new production technology. These production and adoption questions are the central issues explored in this paper. We consider "vertical" coproducts that provide the same function but differ in their performance quality (e.g., speed of microprocessors and strength of diamond) and "horizontal" coproducts that differ in their application and are therefore sold in different markets (e.g., chlorine and sodium hydroxide, acetone and phenol).

Focusing first on a vertical market model in which the two coproducts differ along a performance dimension, we characterize the optimal pricing and production decisions of a monopoly firm with two technologies facing a deterministic market size. We prove that dual activation in which the firm produces a positive quantity on both technologies can be optimal, but only if the technologies differ in their splits. For dual activation to be optimal, one technology must have a lower marginal cost for total product, e.g., cost per good device (slow or fast) in semiconductors, but a higher marginal cost for the higher quality product, e.g., cost per high speed device. The fact that each technology offers a different marginal-cost benefit can make dual activation attractive, but this alone does not guarantee dual activation: single activation can be optimal even when neither technology dominates the other from a marginal cost perspective.

Building on the optimal production analysis, we characterize the optimal adoption-and-usage strategy of a firm with an incumbent technology considering a new technology. Among other implications, we establish that a new technology with an identical split to the incumbent will be adopted if and only if (iff) its effective (i.e., accounting for capacity) yield-adjusted cost is lower than the incumbent's yield-adjusted cost and, if adopted, the new technology displaces the incumbent. If the technologies have the same (effective) yield-adjusted costs, then the new technology is adopted iff it has a higher split of the higher-quality product and it always displaces the incumbent if adopted. When the new technology differs in split and (effective) yield-adjusted cost, then the new technology displaces the incumbent at low effective yield-adjusted costs, is used with the incumbent at intermediate costs, and is not adopted at higher costs. When market size is uncertain at the time of technology adoption, the optimal adoption-and-usage results for the special cases of identical split or identical costs remain unchanged if no uncertainty is resolved until after production. However, if uncertainty is resolved between adoption and production then, because of the risk of unused new capacity, it may be optimal in these special cases to adopt the new technology and use it with the incumbent technology.

Turning next to a horizontal market model in which the two coproducts serve different markets, we again characterize the firm's optimal pricing and production decisions and the resulting optimal technology adoption-and-usage strategy. We show that there is an additional motive for dual activation in the horizontal setting: activating both technologies can help the firm achieve a combined technology split that better resembles the relative market sizes. Among other implications, this gives rise to the result that—different to the vertical setting—if the technologies have the same (effective) yield-adjusted costs, then the new technology may not displace the incumbent if adopted; it can be optimal to use both technologies together.

The rest of the paper is organized as follows. The most relevant literature is discussed in Section 2. The dual-technology production and pricing model is

described in Section 3. The vertical market setting is examined in Section 4 and the horizontal setting in Section 5. The model and analysis are extended to random market sizes in Section 6. Conclusions are presented in Section 7. Proofs are contained in the appendix. Certain additional materials can be found in the online appendix.

2. Literature Review

For the most part, the coproduct literature has focused on operations decisions and to some extent on marketing ones. Motivated by vertical coproduct settings, much of the early coproduct literature, e.g., Bitran and Gilbert (1994), Nahmias and Moinzadeh (1997), Gerchak et al. (1996), focused on the production and downward substitution decisions under split uncertainty.[2] Other vertical coproduct papers have examined pricing (Min and Oren 1996, Tomlin and Wang 2008, Bansal and Transchel 2014) and product-line design (Min and Oren 1996, Chen et al. 2013, Transchel et al. 2016). A number of papers examine procurement and production decisions for horizontal coproducts in the presence of input and/or output spot markets and fixed coproduct splits (Boyabatlı et al. 2011, 2017; Dong et al. 2014; Boyabatlı 2015). Although the end products are horizontal in Boyabatlı et al. (2011) and Dong et al. (2014), both papers allow for an intermediate processing step in which one coproduct can be converted into the other. Different to the above papers, Sunar and Plambeck (2016) focus on how process emissions should be allocated among coproducts when emissions are taxed for environmental reasons.

Our two-technology model adds to the coproduct literature, which, with some exceptions, assumes a single technology. In an extension to their single-technology production and downward substitution model, Gerchak et al. (1996) numerically examine a setting in which the firm has a second technology that only produces the low-quality product. Ng et al. (2012) develop a robust-optimization algorithm for the production and downward substitution problem in the presence of multiple inputs that differ in their coproduct splits, and then test the algorithm performance in a number of computational studies. Boyabatlı et al. (2011) consider spot and contract purchases of beef and assume that the premium-product split is higher for contract purchases. In their model, the firm can process both sources, but it will do so only after all its contract supply has been consumed. In essence, the motive for processing both inputs lies entirely in the potentially limited quantity of the more desirable input. Extending their single-input, oil-refining model to allow the refinery to process inputs that differ in their splits, Dong et al. (2014) establish some properties of the multiple-input problem that allows them to efficiently search for the optimal input portfolio. Our paper completely

characterizes a two-technology, deterministic, coproduct production-and-pricing problem in both horizontal and vertical settings and with technologies that can differ arbitrarily in cost, yield, and split. This allows us to study process development and adoption for coproduct technologies.

Process development and adoption has not been a focus of the coproduct literature. Chen et al. (2013) provide some limited treatment of process improvement in a single-technology, vertical coproduct setting, and find that the firm's profit increases as the output distribution becomes first-order stochastically larger or more variable in a mean-preserving spread sense. As part of their extensive exploration of downward substitution and product withholding, i.e., offering only the high-quality product and not the low one, Bansal and Transchel (2014) numerically explore the impact of the high-quality split on these decisions, finding (among other observations) that downward substitution is of limited benefit when the split is low. Noting that the split can improve over time in the semiconductor industry, they relate this finding to the firm's operations strategy and discuss the implication that downward substitution should increase over the technology's lifecycle. Process development and adoption is not the focus in either of the above papers and, therefore, both assume the firm has only one technology. In contrast, we focus on process development and adoption (in both vertical and horizontal settings) by considering a firm with two coproduct technologies that can differ in cost, overall yield, and split.

Process development and technology adoption have been studied in non coproduct settings. Viewing process development as a costly action that improves a particular process attribute, e.g, cost, quality, capacity, or yield, a stream of papers in operations has explored how a firm operating a single technology (producing a single product) should invest in process improvement efforts over time (Fine and Porteus 1989, Marcellus and Dada 1991, Li and Rajagopalan 1998, Carrillo and Gaimon 2000, Terwiesch and Bohn 2001). Our paper explores a fundamentally different question. Instead of exploring the improvement investment path, we consider a firm with two technologies that can differ in multiple attributes so as to develop insights into technology adoption and usage. Technology adoption and usage relates to technology choice, a topic that has been explored in various settings; for example, whether to invest in flexible and/or dedicated technologies, e.g., Van Mieghem (1998), and whether to invest in environmentally friendly and/or standard technologies, e.g., Wang et al. (2013), Drake et al. (2015).

In closing, we note that a number of sustainability initiatives lead to coproduct type problems. For example, recycling can create streams of commingled materials

requiring separation. The returns used in remanufacturing can vary in quality and therefore require grading into different categories (Ferguson et al. 2006). By-product synergy, in which waste from production of a primary product can be converted into a valuable by-product, (Lee 2012, Zhu et al. 2014, Lee and Tongarlak 2017) creates a coproduct technology.

3. The Production and Pricing Model

We now present the underlying model used in the paper, that being a two-product firm with two coproduct technologies deciding on technology production quantities and coproduct prices. As well as being an important and relevant problem in its own right (because some firms operate a portfolio of coproduct technologies[3]), this production and pricing model serves as our foundation for exploring technology adoption.

The firm produces and sells two coproducts $n = 1, 2$ in a single time period. It has two technologies, labeled A and B, each of which produces the coproducts. The firm determines the selling price $p_n \geq 0$ for each product n and the production quantity $q_t \geq 0$ for each technology $t \in \{A, B\}$, where q_t denotes the number of base units processed, e.g., wafer starts for semiconductors or tonnes of salt for the chloralkali process.[4] Technology t has a marginal (per base unit) production cost of $c_t > 0$, an overall yield per base unit of $y_t > 0$, and a product-2 split of $0 \leq \alpha_{t2} \leq 1$, which implies a product-1 split of $\alpha_{t1} = 1 - \alpha_{t2}$. Therefore, a production pair $\mathbf{q} = (q_A, q_B)$ creates a quantity $Q_n(\mathbf{q}) = \sum_{t \in \{A, B\}} \alpha_{tn} y_t q_t$ of product n at a total cost of $C(\mathbf{q}) = \sum_{t \in \{A, B\}} c_t q_t$. Without loss of generality (w.l.o.g.) we label the technologies such that $\alpha_{A2} \geq \alpha_{B2}$.

Let $R(\mathbf{Q}, \mathbf{p})$ denote the revenue that the firm generates for any given product-quantity pair $\mathbf{Q} = (Q_1, Q_2)$ and price pair $\mathbf{p} = (p_1, p_2)$. This revenue function will depend on the product market in which the firm operates. We assume a monopoly firm and consider two different types of product markets: vertical and horizontal. In both cases, the salvage value of unsold product is set to zero.

In the *vertical-market model*, the two coproducts serve the same basic function but differ along a quality dimension, e.g., semiconductor speed, for which all customers agree that more is better (not accounting for price). Product n has a quality level of x_n, and products are indexed in order of increasing quality, i.e., $x_1 < x_2$. We adopt a standard customer-demand model from the vertical-quality, product-line design literature. There is a deterministic population, size S, of infinitesimal customers that vary in their valuation of quality. Customers are defined by their quality valuation $0 \leq \theta \leq 1$ such that a customer of type θ derives a net utility of $\theta x_n - p_n$ from purchasing a product of quality x_n at price p_n and derives a net utility of 0 if it

does not purchase a product. The firm does not observe a customer's type. Customer types are uniformly distributed between 0 and 1, that is, the proportion of the customer population with a quality valuation less than or equal to θ is given by θ. Similar to Chen et al. (2013), we assume that customers make their first-choice purchase decisions simultaneously, and, in the event that demand exceeds supply of a product, the firm may (i) fill a customer's product demand with a higher-quality product at the lower-quality product's price or (ii) let a customer spill down to its next-choice lower-quality product. All results in the paper apply even if downward fulfillment and/or spill down are not allowed. In this vertical model, the revenue function can be obtained by tailoring Proposition 1 and Equation (6) in Chen et al. (2013) to the case of two products, uniformly distributed customer types, and a population of size S. We then have

$$R(\mathbf{Q}, \mathbf{p}) = p_1 \min\left\{ Q_1 + Q_2, S\left(1 - \frac{p_1}{x_1}\right)^+ \right\}$$
$$+ (p_2 - p_1) \min\left\{ Q_2, S\left(1 - \frac{p_2 - p_1}{x_2 - x_1}\right)^+ \right\}, \quad (1)$$

where $(y)^+ = \max\{y, 0\}$. We note that the product-1 quantity Q_1 influences revenue only through the total product quantity $Q_1 + Q_2$ because the products are vertically differentiated and product 2 is the higher-quality product.

In the *horizontal-market model*, the firm's products serve different applications. For example, chlorine and sodium hydroxide (coproducts of the chloralkali process) have different purposes. Horizontal coproducts are not substitutable from a customer perspective and, therefore, their demands have no cross-price elasticities. We adopt a linear demand function for each coproduct $n = 1, 2$, parameterized as $D_n(p_n) = S_n - (S_n/x_n)p_n$, where $S_n > 0$ is the maximum demand, i.e., the demand at zero price, and $x_n > 0$ is the choke price, i.e., the price at which demand first reaches zero. In this horizontal model, the revenue function is given by

$$R(\mathbf{Q}, \mathbf{p}) = p_1 \min\left\{ \left(S_1 - p_1\left(\frac{S_1}{x_1}\right)\right)^+, Q_1 \right\}$$
$$+ p_2 \min\left\{ \left(S_2 - p_2\left(\frac{S_2}{x_2}\right)\right)^+, Q_2 \right\}. \quad (2)$$

To summarize the firm's pricing and production problem, it chooses coproduct prices $\mathbf{p} = (p_1, p_2)$ and technology production quantities $\mathbf{q} = (q_A, q_B)$ to maximize its profit $\Pi(\mathbf{q}, \mathbf{p}) = R(\mathbf{Q}(\mathbf{q}), \mathbf{p}) - c_A q_A - c_B q_B$, where the quantity of product-n produced is $Q_n(\mathbf{q}) = \sum_{t \in \{A, B\}} \alpha_{tn} y_t q_t$ for $n = 1, 2$, and the revenue function $R(\cdot, \cdot)$ is given by (1) for the vertical market and (2) for the horizontal market. We note that a technology is never used (even if it is the only available technology)

if $c_t/y_t \geq x_1 + (x_2 - x_1)\alpha_{t2}$, and so we assume $c_t/y_t < x_1 + (x_2 - x_1)\alpha_{t2}$ for all technologies throughout the paper. In Section 6, we extend the model to allow for stochastic market sizes.

4. Vertical Coproducts

We first explore and characterize the optimal pricing and production strategy for vertical coproducts. Building on that analysis, we then explore the resulting process development and technology adoption implications.

4.1. Pricing and Production

Recall that the two products are indexed such that product 2 is the higher-quality product, i.e., $x_2 > x_1$, and that the revenue as a function of product prices and product quantities is given by (1). For any non-negative product quantity pair $\mathbf{Q} = (Q_1, Q_2)$, we can adapt the supply constrained, vertically differentiated product, optimal pricing result from Chen et al. (2013) (specifically Theorem 1) for our setting to obtain the following optimal prices:

$$p_{V1}^* = x_1 \max\left\{\frac{1}{2}, 1 - \frac{Q_1 + Q_2}{S}\right\}, \tag{3}$$

$$p_{V2}^* = p_{V1}^* + (x_2 - x_1)\max\left\{\frac{1}{2}, 1 - \frac{Q_2}{S}\right\}, \tag{4}$$

and associated revenue

$$R_V^*(Q_1, Q_2)$$
$$= \begin{cases} x_1\left(1 - \dfrac{Q_1 + Q_2}{S}\right)(Q_1 + Q_2) + (x_2 - x_1)\left(1 - \dfrac{Q_2}{S}\right)Q_2, \\ \qquad\qquad\qquad\qquad Q_1 + Q_2 < \dfrac{S}{2} \\ \dfrac{x_1}{4}S + (x_2 - x_1)\left(1 - \dfrac{Q_2}{S}\right)Q_2, \quad Q_2 < \dfrac{S}{2} \leq Q_1 + Q_2, \\ \dfrac{x_2}{4}S, \qquad\qquad\qquad Q_2 \geq \dfrac{S}{2}. \end{cases} \tag{5}$$

In the region where the total product quantity is less than half of the market size, i.e., $Q_1 + Q_2 < S/2$, the optimal prices are set to exactly match product demands with supplies, and the revenue is increasing in both product quantities. In the "volume saturation" region, i.e., $Q_2 < S/2 \leq Q_1 + Q_2$, the optimal prices are set to clear product 2 but not product 1, and the total product quantity (1 and 2) sold is always $S/2$. The revenue is constant in the product-1 quantity but increasing in the product-2 quantity because more product 2 enables the firm to sell more of the higher-quality product 2 (which commands a higher price) and less of the lower-quality product 1. In the "revenue saturation" region, i.e., $Q_2 \geq S/2$, the firm has an ample supply of product 2 and it is optimal to only sell this higher-quality product. It always sells the quantity $S/2$ at a price of $x_2/2$,

and so the revenue is constant in both product quantities in this region. That being the case, it is never optimal for the firm to set production quantities such that the resulting product supplies lie within this revenue-saturation region.

With the optimal pricing and revenue result in hand, and recalling that a production pair $\mathbf{q} = (q_A, q_B)$ creates a quantity $Q_n(\mathbf{q}) = \sum_{t \in \{A,B\}} \alpha_{tn} y_t q_t$ of product $n = 1, 2$, the firm's pricing and production-quantity problem can be transformed to the following production-quantity problem:

$$\Pi_V^* = \max_{\mathbf{q} \geq \mathbf{0}}\left\{R_V^*\left(\sum_{t \in \{A,B\}}(1 - \alpha_{t2})y_t q_t, \sum_{t \in \{A,B\}}\alpha_{t2}y_t q_t\right)\right.$$
$$\left. - \sum_{t \in \{A,B\}} c_t q_t\right\}, \tag{6}$$

where $R_V^*(\cdot, \cdot)$ is given by (5) and we have used the fact that $\alpha_{t1} = 1 - \alpha_{t2}$, i.e., product splits sum to 1.

It is helpful to first briefly consider the setting in which the firm has only one technology, and so we suppress the technology notation t. The marginal cost of producing good product, i.e., product 1 and product 2 combined, is the "yield-adjusted cost" c/y, e.g., the cost per semiconductor device is the processing cost per wafer start divided by the number of good devices per wafer start. Accounting for its split α_2, the marginal cost of producing the high-quality product is $c/(\alpha_2 y)$.

Proposition 1 (Single Technology). *The optimal production quantity and profit are*

$$q_V^* = \frac{S}{2y}\frac{1}{\alpha_2}\left(1 - \frac{c/y}{\alpha_2(x_2 - x_1)}\right);$$
$$\Pi_V^* = \frac{S}{4}\left(x_1 + (x_2 - x_1)\left(1 - \frac{c/y}{\alpha_2(x_2 - x_1)}\right)^2\right)$$

if $c/y \leq \bar{c}_{\text{sat}}(\alpha_2) \overset{\text{def}}{=} (x_2 - x_1)(1 - \alpha_2)\alpha_2$. *Otherwise*

$$q_V^* = \frac{S}{2y}\frac{x_1 + (x_2 - x_1)\alpha_2 - c/y}{x_1 + (x_2 - x_1)\alpha_2^2};$$
$$\Pi_V^* = \frac{S}{4}\frac{(x_1 + (x_2 - x_1)\alpha_2 - c/y)^2}{x_1 + (x_2 - x_1)\alpha_2^2}.$$

The threshold cost $\bar{c}_{\text{sat}}(\alpha_2)$ determines whether the yield-adjusted cost is low enough to justify producing enough so that the resulting total product supply lies in the volume saturation region. For brevity later, we will say a technology operates in the volume-saturation region if $c/y \leq \bar{c}_{\text{sat}}(\alpha_2)$.

We now return to the two-technology setting. In what follows, we will say a technology is activated if the firm produces a strictly positive quantity using that technology.

Definition 1 (Dominant Technology—Vertical Coproducts). Let $i \in \{A, B\}$ denote one technology and j the other. Technology i dominates technology j iff $c_i/y_i \leq c_j/y_j$ (that is, i has a lower marginal cost for producing good product) and $c_i/(\alpha_{i2}y_i) \leq c_j/(\alpha_{j2}y_j)$ (that is, i has a lower marginal cost for producing the high-quality product). Strict dominance occurs iff at least one inequality is strict.

Intuitively, the firm should not activate a strictly dominated technology because the other technology is cheaper from both a combined product and a high-quality product perspective. More formally, any production pair with a positive quantity of the dominated technology can be adjusted (by reducing the dominated technology quantity and increasing the dominant technology quantity) in a manner that strictly decreases the production cost while at the same time weakly increasing both the supply of product 2 and the supply of overall product (1 and 2); therefore, revenue can be weakly increased and cost strictly decreased. This is formally proved in Lemma 1 in the appendix. In the weak dominance case, both technologies are effectively identical and the firm is indifferent between them.

The following theorem gives the optimal technology activation strategy in general when the firm has two technologies. Recall that (w.l.o.g.) the technologies are labelled such that $\alpha_{A2} \geq \alpha_{B2}$.

Theorem 1 (Two Technologies). (i) *If the technologies have identical product-2 splits, i.e., $\alpha_{A2} = \alpha_{B2}$, then it is optimal to activate only Technology A iff $c_A/y_A \leq c_B/y_B$ and to activate only Technology B otherwise.*

(ii) *If the technologies differ in their product-2 splits, i.e., $\alpha_{A2} > \alpha_{B2}$, then define*

$$\bar{c}_L = \begin{cases} \dfrac{\alpha_{B2}}{\alpha_{A2}} \cdot \dfrac{c_A}{y_A}, & c_A/y_A \leq (x_2 - x_1)(1 - \alpha_{B2})\alpha_{A2} \\ \begin{aligned} &\left[(x_1 + (x_2 - x_1)\alpha_{B2}^2)(c_A/y_A) \right. \\ &\left. \quad - x_1(x_2 - x_1)(\alpha_{A2} - \alpha_{B2})(1 - \alpha_{B2})\right] \\ &\quad \cdot \left[x_1 + (x_2 - x_1)\alpha_{A2}\alpha_{B2}\right]^{-1}, \\ &\qquad \textit{otherwise.} \end{aligned} \end{cases} \quad (7)$$

$$\bar{c}_H = \begin{cases} \dfrac{\alpha_{B2}}{\alpha_{A2}} \cdot \dfrac{c_A}{y_A}, & c_A/y_A \leq (x_2 - x_1)(1 - \alpha_{A2})\alpha_{A2} \\ \begin{aligned} &\left[(x_1 + (x_2 - x_1)\alpha_{A2}\alpha_{B2})(c_A/y_A) \right. \\ &\left. \quad - x_1(x_2 - x_1)(\alpha_{A2} - \alpha_{B2})(1 - \alpha_{A2})\right] \\ &\quad \cdot \left[x_1 + (x_2 - x_1)\alpha_{A2}^2\right]^{-1}, \\ &\qquad \textit{otherwise.} \end{aligned} \end{cases} \quad (8)$$

If $c_B/y_B \leq \bar{c}_L$ then activate only Technology B. If $\bar{c}_L < c_B/y_B < \bar{c}_H$ then activate both Technologies A and B. If $c_B/y_B \geq \bar{c}_H$ then activate only Technology A.[5]

There always exists a dominant technology when both technologies have the same product-2 split, and

single technology activation is therefore optimal in this special case. Single activation is not necessarily optimal when the technologies differ in their product-2 splits. Figure 1 illustrates the optimal activation strategy in general, i.e., Theorem 1(ii), as a function of the technologies' yield-adjusted costs when their splits differ. Recall that the technologies are labeled such that $\alpha_{A2} > \alpha_{B2}$. There always exists a region of yield-adjusted cost pairs for which neither technology dominates; this is the region between the two rays emanating from the origin. In this nondominated region, Technology A is cheaper from a product-2 perspective but Technology B is cheaper from a combined product perspective. Observe that there exists a subregion within the nondominated region for which dual activation is optimal. The area of the dual activation region is given by $A_D = 0.5x_1(x_2 - x_1)(\alpha_{A2} - \alpha_{B2})^2$ (proof omitted). All else held constant, this area increases as the technology split difference increases. Put differently, the larger the split difference between the two technologies, the more cost pairs there are for which dual activation is optimal.

The dual-activation region does not encompass the entire nondominated region. The abbreviated intuition is as follows. Different to the case when one technology dominates, there is no intertechnology production exchange rate that decreases the overall production cost while weakly increasing the supplies of the combined products and of product 2. In other words, when neither technology dominates, production exchange can reduce cost but only by also reducing some product supply and thus revenue. This provides a motive to activate both technologies. However, if the yield-adjusted costs are sufficiently different then there exists an exchange rate that always reduces cost more than it decreases revenue, and so single activation can be optimal in the nondominated region. A more detailed explanation is provided in the following two paragraphs for the interested reader.

Observe that dual activation never occurs when Technology A's yield-adjusted cost is lower than its volume-saturation cost threshold, i.e., $c_A/y_A < (x_2 - x_1) \cdot \alpha_{A2}(1 - \alpha_{A2})$ but dual activation can occur at higher Technology A yield-adjusted costs. Focusing on the low-cost case first, an optimal production pair must lead to volume saturation when $c_A/y_A < (x_2 - x_1)\alpha_{A2}(1 - \alpha_{A2})$ because Technology A's low yield-adjusted cost justifies producing a total product supply that exceeds half the market size. Because total product volume is saturated, the optimal revenue depends only on the product-2 supply. Consider some production vector that uses both technologies. Because Technology A is cheaper than B for making product 2, by an appropriate exchange of B for A we can reduce the overall production cost while both maintaining a constant supply of product 2 and ensuring the total supply still exceeds $S/2$ (and thus

Figure 1. Optimal Technology Activation as a Function of Yield-Adjusted Costs c_A/y_A and c_B/y_B
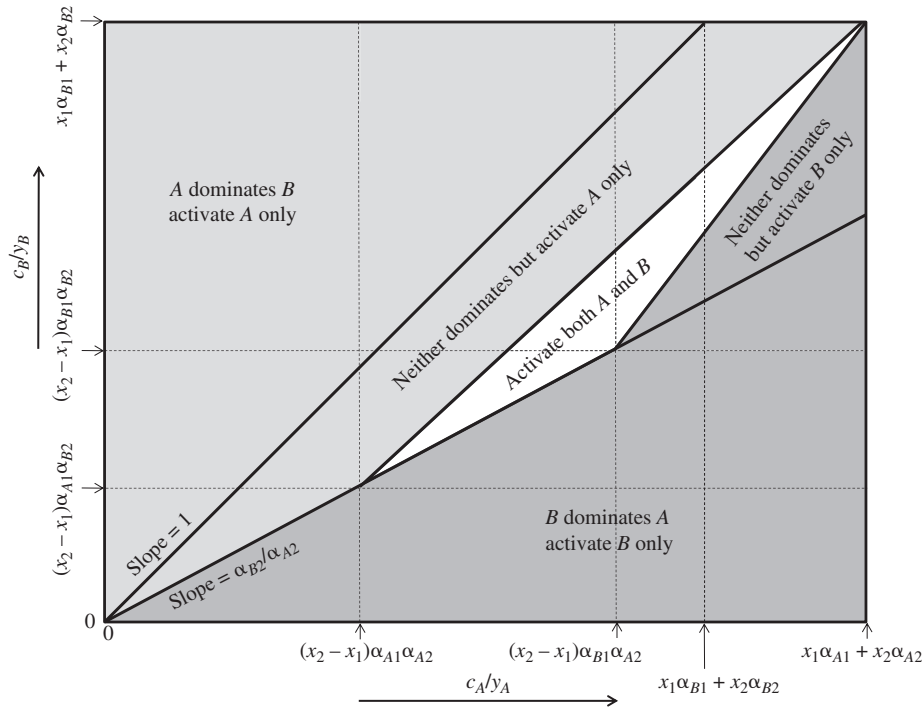
Note. Recall that Technology A has a higher product-2 split than Technology B, i.e., $\alpha_{A2} > \alpha_{B2}$.

maintaining the same revenue). Therefore, no production vector that activates both technologies can be optimal. The only question is which technology should be used. Technology A is the optimal one because it has the lower cost of producing product 2 and because its cost lies below its saturation threshold.

At higher Technology A yield-adjusted costs, i.e., $c_A/y_A \geq (x_2 - x_1)\alpha_{A2}(1 - \alpha_{A2})$, it cannot be optimal to activate Technology A and produce product quantities that lie in the volume saturation region. Therefore, if A is activated, the revenue depends on the total product quantity and the product-2 quantity. Consider some production vector that activates both technologies. Because Technology A is cheaper than B for making product 2, we can reduce the overall production cost while maintaining a constant supply of product-2 by an appropriate (not one-for-one) exchange of B for A, but we lose some revenue by doing this because we strictly reduce the total product supply (1 and 2). This exchange (B for A) is attractive if the associated revenue loss is not too large. The revenue loss becomes more significant as Technology B's production is driven lower because revenue is concave in the total product supply. If Technology B's yield-adjusted cost is high enough relative to A's then it can be optimal to drive Technology B production all the way to zero. Alternatively, because Technology B is cheaper than A for making total product, we can reduce the overall production cost while maintaining a constant supply of total product by an appropriate (not one-for-one)

exchange of A for B, but we lose some revenue by doing this because we strictly reduce product-2 supply. This exchange (A for B) is attractive if the associated revenue loss is not too large. The revenue loss becomes more significant as Technology A's production is driven lower because revenue is concave in product-2 supply. If Technology A's yield-adjusted cost is high enough relative to B's then it can be optimal to drive Technology A production to zero. Therefore, dual activation is optimal when neither technology's yield-adjusted cost is too high relative to other technology, but single technology activation is optimal otherwise. The relevant cost thresholds depend on the product qualities x_1 and x_2 and the technology splits α_{A2} and α_{B2} because these factors influence the marginal revenue loss and the production "exchange rates" required to preserve product-2 supply or total supply.

4.2. Implications for Process Development and Technology Adoption

With the optimal production strategies analyzed, we now turn our attention to the resulting implications for process development and technology adoption.

First let us consider a firm with a single technology looking to improve this technology. It follows from Proposition 1 that the optimal profit decreases in the technology's processing cost c, increases in the technology's yield y, and increases in the technology's product-2 split α_2. Therefore, from a process development perspective, the firm has three different and

distinct directions to pursue: cost reduction, yield increase, or split-2 increase. If the firm can move one of these drivers in the appropriate direction without negatively affecting the other two drivers, then this is an unambiguous improvement. Oftentimes, however, process development may involve trade-offs between the drivers. Let us first consider cost and yield while holding the split constant. A yield-increase initiative may result in a higher processing cost. For example, increasing semiconductor yield by moving to a larger wafer size increases the yield per base-unit processed but also increases the cost per base-unit processed. As the reader may have noticed, cost c and yield y influence the profit only through the yield-adjusted cost c/y; and so a process development initiative that simultaneously alters both cost and yield is an improvement (at a constant split) iff the yield-adjusted cost is decreased. Now, taking the split into account, a process development initiative that increases the split results in a superior technology (i.e., one with a higher profit) only if the associated yield-adjusted cost increase is not too large. In the supplement to this paper, we characterize the allowable cost increase by specifying the iso-profit curve that traces the yield-adjusted cost needed to maintain a constant profit as the product-2 split changes over the range 0 to 1.

Next let us consider a firm with a single incumbent technology evaluating some new technology that differs in one or more of the cost, yield, and split attributes. For example, as discussed in the introduction, a new technology might be an enhanced process technology in the case of LEDs or a different capsule technology in the case of industrial diamonds. When evaluating a new technology (i.e., should it be adopted?), the firm must consider the adoption cost and how the new technology will be used if adopted, i.e., will it displace the incumbent or be used with the incumbent. To explore these adoption-and-usage questions, we consider a firm with one incumbent technology, labeled I, that can adopt a new technology, labeled N. We assume the firm has ample capacity for the incumbent technology but that it must install (or retrofit) capacity if it adopts the new technology. We assume the adoption cost is linear in the capacity of Technology N installed, with $k_N \geq 0$ denoting the capacity cost per base unit.

In this setting, the firm will install a capacity level for Technology N equal to its planned processing quantity (zero if not adopted), and therefore the adoption-and-usage problem is essentially a two-technology production problem in which the new technology's effective yield-adjusted cost includes the capacity cost; that is, the new technology's marginal cost of good product is $(c_N + k_N)/y_N$. The new technology can have a lower or higher split than the incumbent and, therefore, our earlier two-technology production result (Theorem 1)

must be applied with care to account for the fact that the new technology might play the role of "A" (higher split) or "B" (lower split). The optimal adoption-and-usage is illustrated in Figure 2 and formally presented as Corollary 1 in the appendix. In general, there are two thresholds (specified in closed form in the proof of Corollary 1) that govern the optimal strategy. The new technology is adopted iff its effective yield adjusted cost is lower than the adoption threshold but the new technology displaces the incumbent only if its effective yield-adjusted cost is sufficiently low, i.e., lower than the displacement threshold. In between these thresholds, the new technology is used with the incumbent technology.

This general structure has some interesting special cases. First consider the case where the new and the incumbent have the same product-2 splits, i.e., $\alpha_{I2} = \alpha_{N2}$. In this case, the optimal adoption-and-usage strategy is very sharp: the new technology is adopted iff its effective yield-adjusted cost is lower than the incumbent's yield-adjusted cost; and, if adopted, the new technology always displaces the incumbent. Next consider the case where the new and the incumbent have the same (effective) yield-adjusted costs, i.e., $c_I/y_I = (c_N + k_N)/y_N$. Again we have a sharp characterization: the new technology is adopted iff its product-2 split is lower than the incumbent's product-2 split; and, if adopted, the new technology always displaces the incumbent. The fact that the new technology is never used with the incumbent in either of these two special cases (i.e., the adoption and displacement thresholds in Figure 2 coincide) stems from the fact that there is always one dominant technology if two technologies differ only in split or only in yield-adjusted cost. Finally, let us consider a special case in which the new and incumbent differ in both split and (effective) yield-adjusted cost. If the incumbent has a low enough yield-adjusted cost to place it in the volume saturation region, i.e., $\bar{c}_I \leq \bar{c}_{\mathrm{sat},I}(\alpha_{I2})$, then the new technology is adopted iff its effective marginal cost of producing product 2 is lower than the incumbent's marginal cost of producing product 2, i.e., $(c_N + k_N)/(\alpha_{N2}y_N) < c_I/(\alpha_{I2}y_I)$. In other words, in this case adoption is driven solely by the (effective) marginal costs of producing the higher-quality product. The new technology might displace or be used with the incumbent in this special case.

A key implication of the optimal adoption-and-usage strategy is that there can be value to a new technology even if it is inferior to the incumbent (where inferior means a lower profit if used as a single technology). This value arises because an inferior technology can complement the incumbent by improving the firm's (effective) marginal cost of producing good product (1 and 2) or the high-quality product (2) such

Figure 2. Optimal Adoption-and-Usage Strategy for a Firm with One Incumbent Technology as a Function of the New Technology's Effective Yield-Adjusted Cost
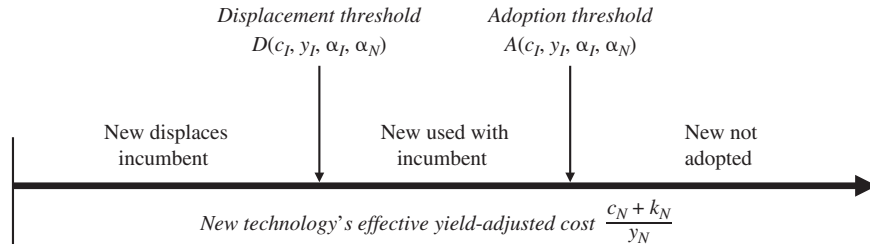
that the new technology falls in the "new-used-with-incumbent" region of Figure 2. The value of an inferior technology can be significant.[6] Of course, inferior technologies do not always add value. Similarly, a new technology that is superior to the incumbent might not displace it; a superior technology can lie in the "new-used-with-incumbent" region.

5. Horizontal Coproducts

We now turn our attention to the horizontal market setting in which the coproducts differ in their application and, therefore, are not substitutable. As for vertical coproducts, we first explore and characterize the optimal pricing and production strategy and then explore the resulting process development and technology adoption implications. Our primary purpose will be to highlight important differences that arise for horizontal coproducts.

5.1. Pricing and Production

Recall from Section 3 that the demand function for product $n = 1, 2$ is defined by its market size S_n (demand when price is zero) and its choke price x_n (price at which demand first reaches zero), that the products are labeled such $x_1 \leq x_2$, and that the revenue as a function of product prices and product quantities is given by (2). It follows that for any product quantities $Q_1 \geq 0$ and $Q_2 \geq 0$, the optimal price of product $n = 1, 2$ is

$$p^*_{Hn} = x_n \max\{1/2, 1 - Q_n/S_n\}, \quad (9)$$

and the resulting total optimal revenue is given by $R^*_H(Q_1, Q_2) = \sum_{n=1}^2 R^*_{Hn}(Q_n)$, where

$$R^*_{Hn}(Q_n) = \begin{cases} x_n(1 - Q_n/S_n)Q_n, & Q_n < S_n/2 \\ S_n x_n/4, & Q_n \geq S_n/2. \end{cases} \quad (10)$$

Different to the vertical-market setting, a product's price and sales volume depend only on its own quantity and are not influenced by the other product's quantity. A product's revenue is saturated iff its supply exceeds half its market size. With the optimal pricing and revenue result in hand, and recalling that a production pair $\mathbf{q} = (q_A, q_B)$ creates a quantity $Q_n(\mathbf{q}) =$

$\sum_{t \in \{A, B\}} \alpha_{tn} y_t q_t$ of product $n = 1, 2$, we obtain the following production-quantity problem:

$$\Pi^*_H = \max_{\mathbf{q} \geq 0} \left\{ R^*_H \left(\sum_{t \in \{A, B\}} (1 - \alpha_{t2}) y_t q_t, \sum_{t \in \{A, B\}} \alpha_{t2} y_t q_t \right) \right.$$
$$\left. - \sum_{t \in \{A, B\}} c_t q_t \right\}, \quad (11)$$

where $R^*_H(\cdot, \cdot)$ is given above.

It is helpful to define $\hat{s}_2 = S_2/(S_1 + S_2)$ as the "market-2 split," i.e., product 2's portion of the combined market sizes, because this will be a key driver of later results. In particular, when technology t's product-2 split is higher (lower) than the market-2 split, i.e., $\alpha_{t2} > \hat{s}_2$ ($\alpha_{t2} < \hat{s}_2$) then technology t's output mix is misaligned with the market mix in the sense that it overproduces (underproduces) product 2 from a proportional perspective. Before addressing the two-technology case, for completeness we first present the optimal production quantity and profit when the firm has a single technology (and so suppress the technology t notation).

Proposition 2 (Single Technology—Horizontal Coproducts). *Define* $S = S_1 + S_2$. *The optimal production quantity and profit are*

$$q^*_H = \frac{S}{2y} \frac{\hat{s}_2}{\alpha_2} \left(1 - \frac{c/y}{\alpha_2 x_2} \right);$$

$$\Pi^*_H = \frac{S}{4} \left((1 - \hat{s}_2) x_1 + \hat{s}_2 x_2 \left(1 - \frac{c/y}{\alpha_2 x_2} \right)^2 \right)$$

if $0 \leq \alpha_2 < \hat{s}_2$ *and* $c/(\alpha_2 y) \leq x_2(1 - (\alpha_2/(1 - \alpha_2)) \cdot ((1 - \hat{s}_2)/\hat{s}_2))$; *are*

$$q^*_H = \frac{S}{2y} \frac{1 - \hat{s}_2}{1 - \alpha_2} \left(1 - \frac{c/y}{(1 - \alpha_2)x_1} \right);$$

$$\Pi^*_H = \frac{S}{4} \left((1 - \hat{s}_2) x_1 \left(1 - \frac{c/y}{(1 - \alpha_2)x_1} \right)^2 + \hat{s}_2 x_2 \right)$$

if $\hat{s}_2 < \alpha_2 \leq 1$ *and* $c/((1 - \alpha_2)y) \leq x_1(1 - ((1 - \alpha_2)/\alpha_2) \cdot (\hat{s}_2/(1 - \hat{s}_2)))$; *and otherwise are*

$$q^*_H = \frac{S}{2y} \frac{x_1 + (x_2 - x_1)\alpha_2 - c/y}{x_1(1 - \alpha_2)^2/(1 - \hat{s}_2) + x_2\alpha_2^2/\hat{s}_2};$$

$$\Pi^*_H = \frac{S}{4} \frac{(x_1 + (x_2 - x_1)\alpha_2 - c/y)^2}{x_1(1 - \alpha_2)^2/(1 - \hat{s}_2) + x_2\alpha_2^2/\hat{s}_2}.$$

The technology's marginal cost of producing product 2 is $c/(\alpha_2 y)$ and the marginal cost of product 1 is $c/((1-\alpha_2)y)$. If the technology output mix is misaligned with the market mix, i.e., $\alpha_2 \neq \hat{s}_2$, and the marginal cost of the overproduced product is low, then the overproduced product is saturated. If the technology output mix is aligned, i.e., $\alpha_2 = \hat{s}_2$, or the marginal costs are not low, then neither product is saturated.

Turning now to the two-technology case, we need to adapt our earlier dominance definition. In the vertical case—because there is one market of customers that choose among the two products based on quality valuations and prices—the relevant marginal costs for dominance are the marginal costs of overall supply (i.e., products 1 and 2) and of the higher-quality product (2) supply. Because product markets are distinct in the horizontal case, dominance depends on the marginal costs of each individual product.

Definition 2 (Dominant Technology—Horizontal Coproducts). Let $i \in \{A, B\}$ denote one technology and j the other. Technology i dominates technology j iff $c_i/((1-\alpha_{i2})y_i) \leq c_j/((1-\alpha_{j2})y_j)$ (that is, i has a lower marginal cost for producing product 1) and $c_i/(\alpha_{i2}y_i) \leq c_j/(\alpha_{j2}y_j)$ (that is, i has a lower marginal cost for producing product 2). Strict dominance occurs iff at least one inequality is strict.

Intuitively, the firm should not activate a strictly dominated technology because the other technology is cheaper for each product, and so revenue can be weakly increased and cost strictly decreased by using only the dominant technology. (See appendix Lemma 2 for proof.)

Before formally presenting the optimal activation strategy when the firm has two technologies, we note that the strategy bears some strong similarities to the vertical coproduct case: single activation is optimal if the splits are identical but dual activation can be optimal when the splits differ and the technology yield-adjusted costs are not too different (accounting for splits). However, in the horizontal case, the boundary costs that specify the dual-activation region depend on how the market mix compares to the technology output mix. In particular, there are three cases: (1) both technologies have product-2 splits greater than the market-2 split, i.e., both overproduce product 2; (2) the technology product-2 splits span the market-2 split; and (3) both technologies have product-2 splits lower than the market-2 split. Moreover, as we discuss below, there is an additional motive for dual activation in the horizontal coproduct case.

Theorem 2 (Two Technologies—Horizontal Coproducts).
(i) *If the technologies have identical product-2 splits, i.e., $\alpha_{A2} = \alpha_{B2}$, then it is optimal to activate only Technology A iff $c_A/y_A \leq c_B/y_B$ and to activate only Technology B otherwise.*

(ii) *If the technologies differ in their product-2 splits, i.e., $\alpha_{A2} > \alpha_{B2}$ (recall $\alpha_{A2} \geq \alpha_{B2}$ w.l.o.g.), then define $\bar{c}_L$ and $\bar{c}_H$ as below. If $c_B/y_B \leq \bar{c}_L$ then activate only Technology B. If $\bar{c}_L < c_B/y_B < \bar{c}_H$ then activate both Technologies A and B. If $c_B/y_B \geq \bar{c}_H$ then activate only Technology A:*[7]

$$\bar{c}_L = \begin{cases} \dfrac{c_A/y_A}{1-\alpha_{A2}}(1-\alpha_{B2}), \\ \qquad c_A/y_A \leq x_1(1-\alpha_{A2})\left(1-\dfrac{\hat{s}_2(1-\alpha_{B2})}{(1-\hat{s}_2)\alpha_{B2}}\right), \\ \left[(\hat{s}_2 x_1(1-\alpha_{B2})^2 + (1-\hat{s}_2)x_2\alpha_{B2}^2)c_A/y_A \right. \\ \qquad \left. + x_1 x_2(\alpha_{A2}-\alpha_{B2})(\alpha_{B2}-\hat{s}_2)\right] \\ \quad \cdot \left[\hat{s}_2 x_1(1-\alpha_{A2})(1-\alpha_{B2}) + (1-\hat{s}_2)x_2\alpha_{A2}\alpha_{B2}\right]^{-1}, \\ \qquad otherwise, \end{cases}$$

$$\bar{c}_H = \begin{cases} \dfrac{c_A/y_A}{1-\alpha_{A2}}(1-\alpha_{B2}), \\ \qquad c_A/y_A \leq x_1(1-\alpha_{A2})\left(1-\dfrac{\hat{s}_2(1-\alpha_{A2})}{(1-\hat{s}_2)\alpha_{A2}}\right), \\ \left[(\hat{s}_2 x_1(1-\alpha_{A2})(1-\alpha_{B2}) + (1-\hat{s}_2)x_2\alpha_{A2}\alpha_{B2})c_A/y_A \right. \\ \qquad \left. + x_1 x_2(\alpha_{A2}-\alpha_{B2})(\alpha_{A2}-\hat{s}_2)\right] \\ \quad \cdot \left[\hat{s}_2 x_1(1-\alpha_{A2})^2 + (1-\hat{s}_2)x_2\alpha_{A2}^2\right]^{-1}, \\ \qquad otherwise, \end{cases}$$

if $\hat{s}_2 \leq \alpha_{B2}$ (Case 1),

$$\bar{c}_L = \begin{cases} \dfrac{c_A/y_A}{\alpha_{A2}}\alpha_{B2}, \quad c_A/y_A \leq x_2\alpha_{A2}\left(1-\dfrac{(1-\hat{s}_2)\alpha_{B2}}{\hat{s}_2(1-\alpha_{B2})}\right), \\ \left[(\hat{s}_2 x_1(1-\alpha_{B2})^2 + (1-\hat{s}_2)x_2\alpha_{B2}^2)c_A/y_A \right. \\ \qquad \left. + x_1 x_2(\alpha_{A2}-\alpha_{B2})(\alpha_{B2}-\hat{s}_2)\right] \\ \quad \cdot \left[\hat{s}_2 x_1(1-\alpha_{A2})(1-\alpha_{B2}) + (1-\hat{s}_2)x_2\alpha_{A2}\alpha_{B2}\right]^{-1}, \\ \qquad otherwise, \end{cases}$$

$$\bar{c}_H = \begin{cases} \dfrac{c_A/y_A}{1-\alpha_{A2}}(1-\alpha_{B2}), \\ \qquad c_A/y_A \leq x_1(1-\alpha_{A2})\left(1-\dfrac{\hat{s}_2(1-\alpha_{A2})}{(1-\hat{s}_2)\alpha_{A2}}\right), \\ \left[(\hat{s}_2 x_1(1-\alpha_{A2})(1-\alpha_{B2}) + (1-\hat{s}_2)x_2\alpha_{A2}\alpha_{B2})c_A/y_A \right. \\ \qquad \left. + x_1 x_2(\alpha_{A2}-\alpha_{B2})(\alpha_{A2}-\hat{s}_2)\right] \\ \quad \cdot \left[\hat{s}_2 x_1(1-\alpha_{A2})^2 + (1-\hat{s}_2)x_2\alpha_{A2}^2\right]^{-1}, \\ \qquad otherwise, \end{cases}$$

if $\alpha_{B2} < \hat{s}_2 < \alpha_{A2}$ (Case 2), and

$$\bar{c}_L = \begin{cases} \dfrac{c_A/y_A}{\alpha_{A2}}\alpha_{B2}, \quad c_A/y_A \leq x_2\alpha_{A2}\left(1-\dfrac{(1-\hat{s}_2)\alpha_{B2}}{\hat{s}_2(1-\alpha_{B2})}\right), \\ \left[(\hat{s}_2 x_1(1-\alpha_{B2})^2 + (1-\hat{s}_2)x_2\alpha_{B2}^2)c_A/y_A \right. \\ \qquad \left. + x_1 x_2(\alpha_{A2}-\alpha_{B2})(\alpha_{B2}-\hat{s}_2)\right] \\ \quad \cdot \left[\hat{s}_2 x_1(1-\alpha_{A2})(1-\alpha_{B2}) + (1-\hat{s}_2)x_2\alpha_{A2}\alpha_{B2}\right]^{-1}, \\ \qquad otherwise, \end{cases}$$

$$\bar{c}_H = \begin{cases} \dfrac{c_A/y_A}{\alpha_{A2}}\alpha_{B2}, & c_A/y_A \le x_2\alpha_{A2}\left(1 - \dfrac{(1-\hat{s}_2)\alpha_{A2}}{\hat{s}_2(1-\alpha_{A2})}\right) \\[2ex] \begin{aligned}[(\hat{s}_2 x_1(1-\alpha_{A2})(1-\alpha_{B2})+(1-\hat{s}_2)x_2\alpha_{A2}\alpha_{B2})c_A/y_A \\ +x_1 x_2(\alpha_{A2}-\alpha_{B2})(\alpha_{A2}-\hat{s}_2)] \end{aligned} \\ \quad \cdot \left[\hat{s}_2 x_1(1-\alpha_{A2})^2 + (1-\hat{s}_2)x_2\alpha_{A2}^2\right]^{-1}, \\[1ex] \qquad\qquad otherwise, \end{cases}$$

if $\hat{s}_2 \ge \alpha_{A2}$ (*Case* 3).

For technologies that differ in their splits, Figure 3 illustrates the optimal activation strategy (in generality) for each of three cases. In each case, there always exists a region of yield-adjusted cost pairs for which neither technology dominates; this is the region between the two rays emanating from the origin. In this nondominated region, Technology A is cheaper from a product-2 perspective but Technology B is cheaper from

a product-1 perspective. The nondominated region is larger in the horizontal setting than the vertical setting because this dominance criterion is more difficult to satisfy. Analogously to the vertical coproduct setting, when neither technology dominates there may not exist a production exchange rate that allows the firm to profitably increase production on one technology while reducing production on the other. Therefore, in all three cases there exists a subregion for which dual activation is optimal. While the dual-activation region differs between the cases, the region's area is always larger in Case 2 than in Cases 1 or 3 (proof omitted). In other words, when the technologies' product-2 splits span the market-2 split, there are more cost pairs for which dual activation is optimal. The reason for this lies in an additional dual-activation motive in the horizontal coproduct setting. If one technology overproduces product 2

Figure 3. Optimal Technology Activation as Function of Yield-Adjusted Costs c_A/y_A and c_B/y_B
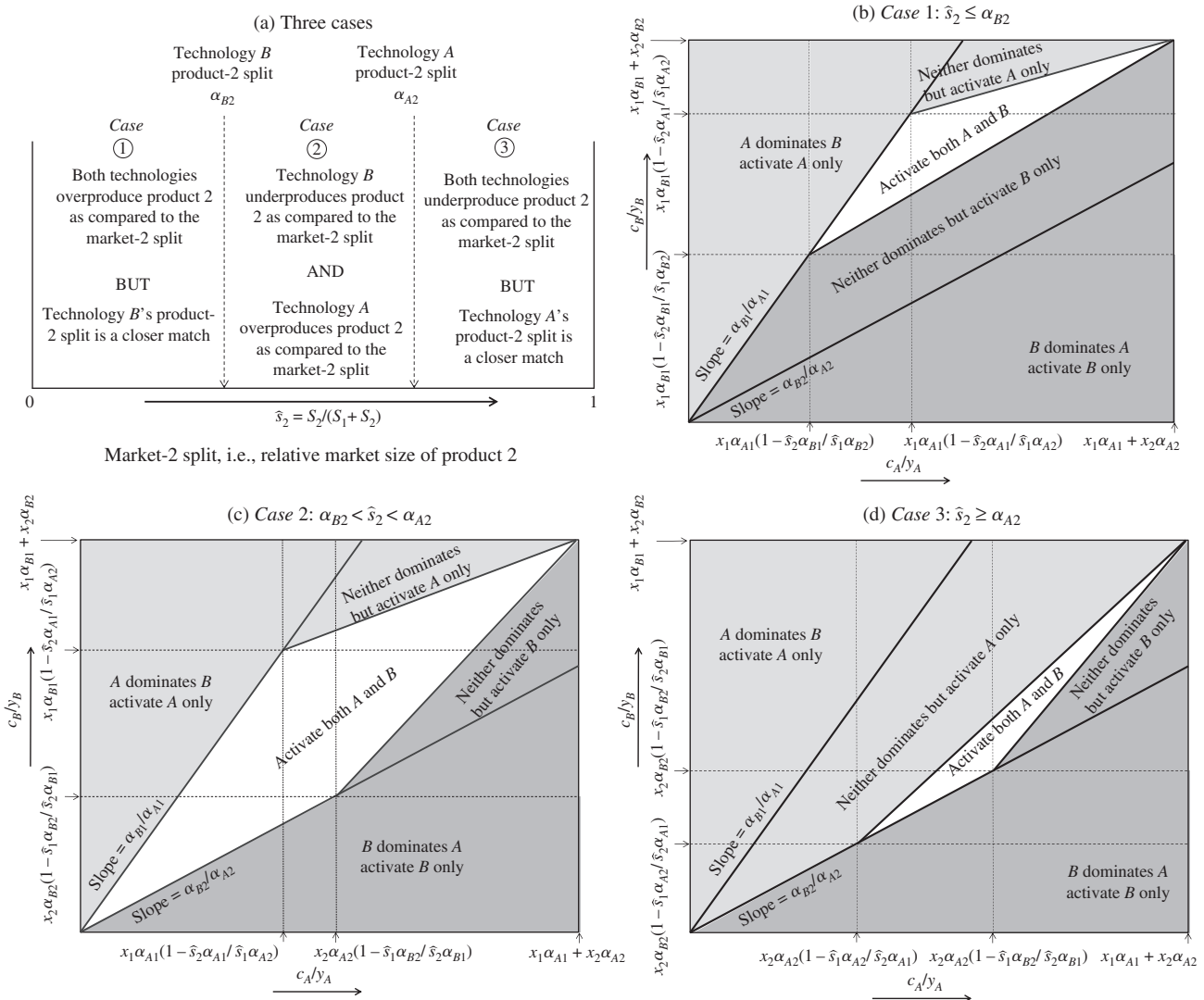
Notes. Recall that Technology B has a higher product-2 split than Technology B, i.e., $\alpha_{A2} > \alpha_{B2}$. Note that $\hat{s}_1 = 1 - \hat{s}_2$.

and the other underproduces product 2 in relation to the market-2 split, i.e., Case 2, then activating both technologies can help the firm achieve a combined technology output mix that better resembles the market mix.

5.2. Implications for Process Development and Technology Adoption

With the optimal production strategies analyzed, we now turn our attention to the resulting implications for process development and technology adoption. Some implications are similar in the horizontal and vertical settings, and in what follows we focus on where the implications diverge rather than presenting and discussing all results.

First, let us consider a firm with a single technology looking to improve this single technology. Analogous to the vertical case, it follows from Proposition 2 that the optimal profit decreases in the technology's processing cost c and increases in the technology's yield y. In fact, as before, the profit increases as the yield-adjusted cost c/y decreases. If product 1's choke price x_1 is less than the technology's yield-adjusted cost c/y, then product 1 is not economically viable in its own right, i.e., it alone cannot justify production. Using accounting terminology, we say that product 1 is a by-product if $x_1 \leq c/y$ and a coproduct, i.e., economically viable in its own right, otherwise. Different to the vertical case, the optimal profit is not necessarily increasing in the product-2 split α_2. In fact, the profit is increasing in α_2 iff $c/y \geq x_1$, i.e., product 1 is a by-product. Otherwise, when product 1 is a coproduct, there exists a product-2 split less than one that attains the highest profit.

Proposition 3 (Ideal Market Product-2 Split). *For any given yield-adjusted cost c/y, the profit is highest at a product-2 split of $\alpha_2^* = 1$ if $c/y \geq x_1$, and at $\alpha_2^* = (x_1 \hat{s}_2 \cdot (x_2 - c/y))/(x_2(1 - \hat{s}_2)(x_1 - c/y) + x_1 \hat{s}_2(x_2 - c/y)) < 1$ otherwise.*

This "ideal" technology split α_2^* equals the market-2 split $\hat{s}_2$ if $x_2 = x_1$ or if $c = 0$. It exceeds the market-2 split otherwise, i.e., $\alpha_2^* \geq \hat{s}_2$. In other words, if process development could control the split (at a given yield-adjusted cost), then it should develop a technology split that overshoots the market-2 split. Different to the vertical setting, however, it should not try and push the technology split all the way to 1, i.e., create a technology that only produces product 2, unless product 1 is a by-product.

Turning to the question of process adoption and usage when the firm has one incumbent technology (labeled I) and is evaluating a new technology (labeled N), the optimal adoption-and-usage strategy has a similar structure to the vertical case (see Figure 2) but the adoption and displacement threshold expressions—given in Corollary S1 of the online appendix—differ. Despite the structural similarity, the

adoption-and-usage strategy for horizontal coproducts exhibits some important differences because of the market split and related concept of an ideal product-2 split. Recall that for vertical coproducts the new technology is never used with the incumbent if they have the same (effective) yield-adjusted costs: the new technology either displaces the incumbent or is not adopted. In contrast, because there exists an ideal split less than one for horizontal coproducts, a new and incumbent technology with the same (effective) yield-adjusted costs may be used together. This is formally established in Corollary 2 in the appendix and illustrated in Figure 4. The firm wants to use the technologies in a manner that achieves a combined product-2 split as close as possible to the incumbent's ideal split α_2^* given in Proposition 3. If the technology product-2 splits span this ideal split, then the firm can exactly attain the ideal split by using both technologies in appropriate proportions.

As discussed for vertical coproducts, the firm can benefit—sometimes significantly—from a new technology that is inferior to the incumbent. Because there is an additional motive for dual activation in the horizontal setting, that being the desire to better match the market-2 split when the technology product-2 splits span the market-2 split, our numerical testing has found that inferior technologies can be especially beneficial in the horizontal setting.[8]

6. Uncertainty in Market Size

We now relax the deterministic market size assumption and examine settings in which the size(s)—S in the vertical model and S_1 and S_2 in the horizontal model—are uncertain. We consider two possible uncertainty-resolution sequences. In the first sequence we assume market size uncertainty is resolved after production decisions are made but before prices are set. In the second sequence, when considering technology adoption, we assume market size uncertainty is resolved after capacity investment but before production and prices are set. For reasons of space, we relegate the analysis and results for both sequences to Section A3 in the online appendix and only summarize our key findings in the main body.

6.1. Market Uncertainty Resolved After Production

We first consider the sequence in which the market size is uncertain (with a continuous distribution) at the time of production but prices are set in recourse. When considering technology adoption, we assume there is no uncertainty resolution between capacity investment and production decisions. As one would anticipate, the firm's optimal expected profit is lower when the market is stochastic; see Lemmas S3 and S6 in Section A3 in the online appendix. We have generalized all our earlier analytical results for the deterministic market to

Figure 4. Different to the Vertical Coproduct Setting, New and Incumbent Technologies with the Same (Effective) Yield-Adjusted Costs May Be Used Together in the Horizontal Coproduct Setting
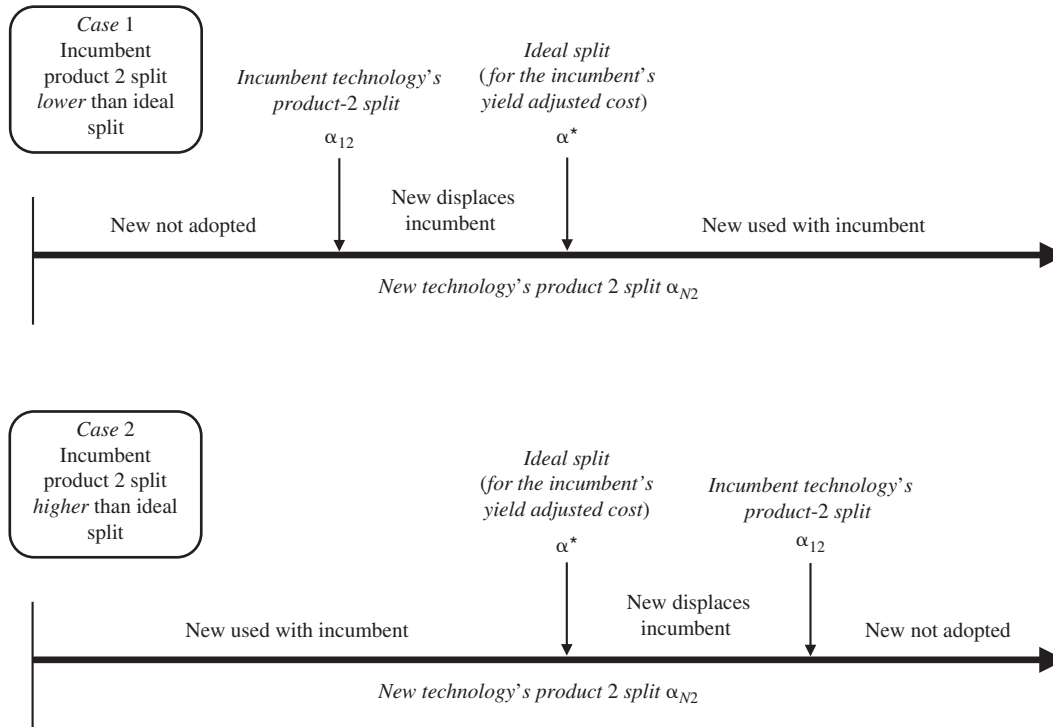
the stochastic market. The analogue to each result can be found in Section A3 in the online appendix. We note that various expressions—such as profits, production quantities, cost thresholds, and ideal split—that have closed-form expressions in the deterministic setting are defined by implicit functions in the stochastic setting. This difference aside, all our earlier production and technology-and-adoption results and associated implications extend (with a minor caveat) to the stochastic market setting.

The caveat relates to the volume- and revenue-saturation product-quantity boundaries in the vertical model (see (5)) and the revenue-saturation product-quantity boundary in the horizontal model (see (10)). When the market size is stochastic with an infinite upper support, there is always a positive probability that any given production quantity will result in product quantities that do not fall in a saturation region. As such, in general there are not sharp cost thresholds—such as the volume saturation cost $c_{sat}(\alpha_2)$ in Proposition 1—that dictate whether an optimal production quantity will result in saturation or not. For the vertical market model, this means that when market size is random, the optimal production quantity on a low-cost technology may not lead to volume saturation in all market size scenarios. In Theorem 1 and the associated Figure 1, we established that dual activation is never optimal if the higher-split technology's yield-adjusted cost was low, i.e., below its volume saturation cost.

When market size is random, dual activation can be optimal even if the yield-adjusted cost is low. This is shown in Figure 5, which depicts the dual activation region for a stochastic market size and a deterministic market size.[9] For Technology A yield-adjusted costs below 0.4 (the volume saturation yield-adjusted cost for Technology A), dual activation is not optimal for the deterministic setting but can be optimal for the stochastic setting. The dual activation region is also shifted somewhat at higher costs.

Finally, we note that this saturation-cost caveat has one implication for our vertical model adoption-and-usage cost findings. In the deterministic setting the adoption rule is particularly simple in the special case when the incumbent technology's yield-adjusted cost is below its volume saturation cost: the new technology is adopted iff it has a lower (effective) marginal cost of producing the higher quality product. Because the volume saturation cost does not exist in the stochastic setting, this special case no longer exists.

6.2. Market Uncertainty Resolved Before Production

In settings where the capacity installation time is long relative to the production lead time, the market size uncertainty at the time of technology adoption is likely to be larger then at the time of production. In what follows, we assume all uncertainty is resolved between capacity investment and production. That is, technology adoption occurs under uncertainty but production

Figure 5. (Color online) Optimal Technology Activation for Stochastic and Deterministic Market Sizes (Vertical Coproducts)
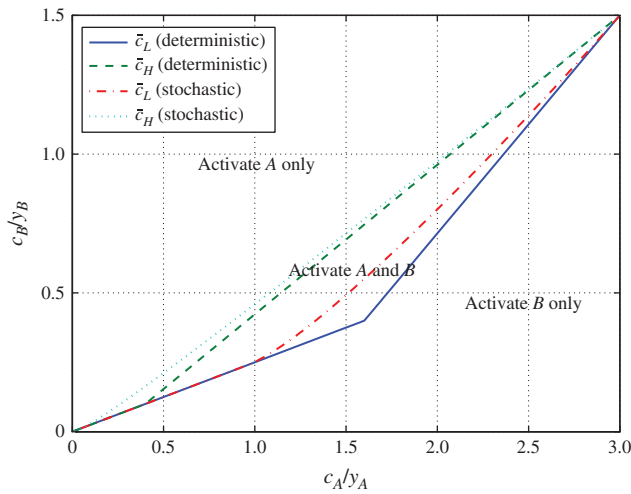
and prices are set in recourse. As before—see Sections 4.2 and 5.2—we consider a firm with an incumbent technology deciding whether or not to adopt a new technology with a linear capacity cost. We assume that the market size(s) has a Bernoulli distribution and furthermore in the horizontal model assume that the product markets are both high or both low. We assume the incumbent technology has sufficient capacity such that it does not limit the optimal production quantity if the market size is high.

In the deterministic market setting, we established for both vertical and horizontal models that the adoption-and-usage strategy is very sharp when the new and the incumbent have the same product-2 splits: the new technology always displaces the incumbent if it is adopted, i.e., they are never used together. This remains true in the stochastic market setting when there is no uncertainty resolution between capacity investment and production. However, a more nuanced strategy emerges when uncertainty is resolved before production. As formally established in Propositions S1 and S3 in Section A3 in the online appendix, there exists a range of incumbent yield-adjusted costs for which the new technology is adopted but it does not displace the incumbent technology: both the new and incumbent are used if the market size is high but only the new is used if the market size is low. The reason that the new technology does not necessarily displace the incumbent when adopted is that there is a risk of unused new technology capacity. Fearing the low-market scenario (when this probability is large enough), the firm does not invest in a sufficient capacity of the new technology that would enable it to produce exclusively on the new technology if the market size turns out to be high. The firm prefers to supplement new technology production with incumbent production if the market size is high rather than invest

in higher capacity levels in advance. This adopt-but-not-displace strategy (when splits are identical) does not arise if there is no uncertainty resolution between capacity and production because there is no capacity risk in that earlier setting: the firm's new technology production level is the same for all market sizes and equals its capacity investment.

Similarly, the earlier finding in the deterministic vertical market that the new and incumbent technology are never used together if technologies have identical effective costs is more nuanced when market size uncertainty is resolved between adoption and production. As with the identical split setting, it can be optimal to adopt the new technology and to then use it with the incumbent when the market size is large; see Propositions S2 and S4 in Section A3 in the online appendix.

7. Conclusions

Processing cost, overall yield, and coproduct split are all important drivers of a coproduct technology's production economics. Firms, therefore, develop new process technologies that differ on one or more of these dimensions so as to improve their underlying economics. Examples (discussed in the introduction) include a lower-cost chloralkali process, a higher yield cumene process, a higher split LED process, larger wafer semiconductor processes that increase yield but increase cost, and a new diamond process that increased yield and split but also cost. The development of new technologies gives rise to the adoption and production questions studied in this paper: Should the firm adopt a new technology and if so should it be used with the incumbent technology or should it displace it?

Focusing first on a vertical market model in which the two coproducts differ along a performance dimension, we characterized the optimal pricing and production decisions of a monopoly firm with two technologies facing a deterministic market size. We proved that dual activation in which the firm produces a positive quantity on both technologies can be optimal if the technologies differ in their splits. Dual activation can only occur when one technology has a lower marginal cost for good product (i.e., both products combined) but the other has a lower marginal cost for the high-quality product. The fact that each technology offers a different marginal-cost benefit can make dual activation attractive. However, this alone does not guarantee dual activation: even if neither technology dominates, single activation can be optimal if the technologies differ sufficiently in their yield-adjusted cost. Building on the optimal production analysis, we characterized the optimal adoption-and-usage strategy of a firm with an incumbent technology considering a new technology. Among other implications, we established that a new

technology with an identical split to the incumbent will be adopted iff its effective (i.e., capacity loaded) yield-adjusted cost is lower than the incumbent's yield adjusted cost and, if adopted, the new technology displaces the incumbent. If the technologies have the same (effective) yield-adjusted costs, then the new technology is adopted iff it has a higher product-2 split and it will displace the incumbent if adopted. When the new technology differs in split and (effective) yield-adjusted cost, then the new technology displaces the incumbent at low effective yield-adjusted costs, is used with the incumbent at intermediate costs, and is not adopted at higher costs. We also explored the impact of market size uncertainty. In the setting where uncertainty is resolved between adoption and production, we established that the risk of unused capacity can induce the firm to adopt the new technology and use it with the incumbent even when their spits are identical.

Focusing next on a horizontal market model in which the two coproducts serve different markets, we characterized the firm's optimal pricing and production decisions and the resulting optimal technology adoption-and-usage decisions. Analogous to the vertical setting, we established that dual activation can be optimal when one technology does not dominate the other, but dominance in the horizontal setting depends on the marginal cost of each individual product. Importantly, we showed that there is an additional motive for dual activation in the horizontal setting: when the technology splits lie on either side of the market split, then activating both technologies can help the firm achieve a combined technology split that better resembles the market split. Among other implications, this gives rise to the result that—different to the vertical setting—if the technologies have the same (effective) yield-adjusted costs it can be optimal to use both technologies together to achieve a more desirable overall product split.

We hope this research opens up other lines of inquiry. We explored a monopoly setting, and other models, e.g., a perfectly competitive or oligopoly market, would shed light on the effect of competition on production and technology adoption. We purposely excluded yield and/or split uncertainty from our model to remove production risk as a driver of dual activation or technology choice. Yield and split can be uncertain in some coproduct settings, both at the adoption and production stages, and it would be of interest to understand the impact of technology risk on production and adoption. Production learning may lead to improvements in cost, yield, and/or split over time, and a multiperiod model could refine some of our findings in settings where such learning is significant. For example, it may be optimal to use both technologies during some transition period until the new technology achieves its long-run cost, yield, and split. Finally,

we note that some coproduct technologies—such as naphtha cracking to produce ethylene and propylene—have the ability to control the product split (within some range) by adjusting the process settings. This gives rise to interesting questions about the value of split control when product markets are uncertain.

Appendix. Proofs
Important Note That Applies to All Proofs
As presented in Section 3, the firm's pricing and production problem is to choose coproduct prices $\mathbf{p} = (p_1, p_2)$ and technology production quantities $\mathbf{q} = (q_A, q_B)$ so as to maximize its profit $\Pi(\mathbf{q}, \mathbf{p}) = R(\mathbf{Q}(\mathbf{q}), \mathbf{p}) - \sum_{t \in \{A, B\}} c_t q_t$, where the quantity of product-n produced is $Q_n(\mathbf{q}) = \sum_{t \in \{A, B\}} \alpha_{tn} y_t q_t$ for $n = 1, 2$, and the revenue function $R(\cdot, \cdot)$ is given by (1) for the vertical market and (2) for the horizontal market. To ensure viability, we assume $c_t / y_t < x_1 + (x_2 - x_1)\alpha_{t2}$ for all technologies throughout. Defining $\bar{c}_t = c_t / y_t$ as the yield-adjusted production cost and $\bar{q}_t = q_t y_t$ as the yield-adjusted production quantity, the above problem is equivalent to $\Pi^* = \max_{\bar{\mathbf{q}} \geq 0, \mathbf{p} \geq 0} \{R(\mathbf{Q}(\bar{\mathbf{q}}), \mathbf{p}) - \sum_{t \in \{A, B\}} \bar{c}_t \bar{q}_t\}$ where $Q_n(\bar{\mathbf{q}}) = \sum_{t \in \{A, B\}} \alpha_{tn} \bar{q}_t$ for $n = 1, 2$. We will use this more notationally compact formulation throughout the proofs. Furthermore, for expositional brevity, we may at times refer to $\bar{c}_t = c_t / y_t$ as a technology's "production cost" and $\bar{q}_t = q_t y_t$ as the technology's "production quantity," with the understanding that these are in fact yield-adjusted costs and yield-adjusted quantities. Actual production quantities given in certain results, e.g., Propositions 1 and 2, are obtained by the conversion $q_t = \bar{q}_t / y_t$.

Proof of Proposition 1. (Vertical Single Technology). Applying (6) to the single-technology case, the firm's problem is to maximize the profit function $\Pi(\bar{q}) = R_V^*((1 - \alpha_2)\bar{q}, \alpha_2 \bar{q}) - \bar{c}\bar{q}$, where $R_V^*(\cdot, \cdot)$ is given by (5). Using (5), we can rewrite $R_V^*((1 - \alpha_2)\bar{q}, \alpha_2 \bar{q})$ as $R_V^*(\bar{q}) = r_1(\bar{q}) + r_2(\bar{q})$, where

$$r_1(\bar{q}) = \begin{cases} x_1\left(1 - \dfrac{\bar{q}}{S}\right)\bar{q}, & \bar{q} \leq S/2 \\ \dfrac{x_1 S}{4}, & \bar{q} > S/2 \end{cases};$$

$$r_2(\bar{q}) = \begin{cases} (x_2 - x_1)\left(1 - \dfrac{\bar{q}\alpha_2}{S}\right)\bar{q}\alpha_2, & \bar{q}\alpha_2 \leq S/2 \\ \dfrac{(x_2 - x_1)S}{4}, & \bar{q}\alpha_2 > S/2. \end{cases}$$

The first and second derivatives of the profit function $\Pi(\bar{q}) = r_1(\bar{q}) + r_2(\bar{q}) - \bar{c}\bar{q}$ are

$$\frac{d\Pi(\bar{q})}{d\bar{q}} = \begin{cases} x_1\left(1 - \dfrac{2\bar{q}}{S}\right) + (x_2 - x_1)\left(1 - \dfrac{2\bar{q}\alpha_2}{S}\right)\alpha_2 - \bar{c}, & \bar{q} \leq \dfrac{S}{2} \\ (x_2 - x_1)\left(1 - \dfrac{2\bar{q}\alpha_2}{S}\right)\alpha_2 - \bar{c}, & \dfrac{S}{2} < \bar{q} \leq \dfrac{S}{2\alpha_2} \\ -\bar{c}, & \bar{q} > \dfrac{S}{2\alpha_2} \end{cases}$$

$$\frac{d^2\Pi(\bar{q})}{d\bar{q}^2} = \begin{cases} x_1\left(-\dfrac{2}{S}\right) + (x_2 - x_1)\left(-\dfrac{2\alpha_2}{S}\right)\alpha_2, & \bar{q} \leq \dfrac{S}{2} \\ (x_2 - x_1)\left(-\dfrac{2\alpha_2}{S}\right)\alpha_2, & \dfrac{S}{2} < \bar{q} \leq \dfrac{S}{2\alpha_2} \\ 0, & \bar{q} > \dfrac{S}{2\alpha_2}. \end{cases}$$

It follows that $\Pi(\bar{q})$ is concave. Furthermore, it is strictly decreasing for $\bar{q} > S/(2\alpha_2)$. The optimal quantity q^* exceeds $S/2$ iff $d\Pi(\bar{q})/d\bar{q} > 0$ at $q = S/2$, or equivalently, $\bar{c} \leq (x_2 - x_1)(1 - \alpha_2)\alpha_2$. Proof of proposition statement follows by application of appropriate first-order condition and subsequent substitution into the profit function. $\square$

Lemma 1 (Vertical Coproducts Dominance). *For $i \in \{A, B\}$ let j denote the other technology, i.e., $j = \{A, B\} \backslash i$. If Technology i dominates Technology j then there exists an optimal production vector $\bar{\mathbf{q}}^*$ that does not use Technology j, i.e., $\bar{q}_j^* = 0$.*

Proof of Lemma 1. (Vertical Coproducts Dominance). We first establish that the optimal revenue $R_V^*(Q_1, Q_2)$ is increasing in product quantities $Q_1 + Q_2$ and Q_2. It is trivially true for $Q_1 + Q_2 \geq S/2$ or $Q_2 \geq S/2$ as $R_V^*(Q_1, Q_2)$ is constant in $Q_1 + Q_2$ and Q_2, respectively. For $Q_1 + Q_2 < S/2$, we have $\partial R_V^*(Q_1, Q_2)/\partial(Q_1 + Q_2) = x_1(1 - 2(Q_1 + Q_2)/S) \geq 0$. Similarly, we have $\partial R_V^*(Q_1, Q_2)/\partial Q_2 = x_1(1 - 2Q_1/S) + x_2(1 - 2Q_2/S) \geq x_1(1 - 2(Q_1 + Q_2)/S) \geq 0$. For $Q_1 + Q_2 \geq S/2$ but $Q_2 < S/2$, we have $\partial R_V^*(Q_1, Q_2)/\partial Q_2 = (x_2 - x_1)(1 - 2Q_2/S) \geq 0$.

From the dominance Definition 1, Technology i dominates technology j iff $c_i/y_i \leq c_j/y_j$ and $c_i/(\alpha_{i2} y_i) \leq c_j/(\alpha_{j2} y_j)$; or equivalently using the yield-adjusted cost notation, Technology i dominates technology j iff $\bar{c}_i \leq \bar{c}_j$ and $\bar{c}_i/\alpha_{i2} \leq \bar{c}_j/\alpha_{j2}$. Noting that $\alpha_{t1} + \alpha_{t2} = 1$ for $t = i, j$, this dominance criterion is equivalent to $\bar{c}_i/\sum_{k=n}^2 \alpha_{ik} \leq \bar{c}_j/\sum_{k=n}^2 \alpha_{jk}$ for $n = 1, 2$.

Consider any arbitrary production vector $\bar{\mathbf{q}}$ with $\bar{q}_j > 0$. Construct a new production vector $\hat{\mathbf{q}}$ that is identical to $\bar{\mathbf{q}}$ except that $\hat{q}_j = 0$ and $\hat{q}_i = \bar{q}_i + (\bar{c}_j/\bar{c}_i)\bar{q}_j$. The resulting output quantities $(\hat{Q}_1 + \hat{Q}_2, \hat{Q}_2)$ are component-wise weakly larger than the quantities $(Q_1 + Q_2, Q_2)$ because $\bar{c}_i/\sum_{k=n}^2 \alpha_{ik} \leq \bar{c}_j/\sum_{k=n}^2 \alpha_{jk} \ \forall n \Rightarrow \sum_{k=n}^2 \alpha_{ik}/\bar{c}_i \geq \sum_{k=n}^2 \alpha_{jk}/\bar{c}_j \Rightarrow \sum_{k=n}^2 \alpha_{ik}(\bar{c}_j/\bar{c}_i)\bar{q}_j \geq \sum_{k=n}^2 \alpha_{jk}\bar{q}_j \Rightarrow \sum_{k=n}^2 \alpha_{ik}(\bar{q}_i + (\bar{c}_j/\bar{c}_i)\bar{q}_j) \geq \sum_{k=n}^2 \alpha_{ik}\bar{q}_i + \sum_{k=n}^2 \alpha_{jk}\bar{q}_j \Rightarrow (\hat{Q}_1 + \hat{Q}_2, \hat{Q}_2) \geq (Q_1 + Q_2, Q_2)$. From above, $R_V^*(Q_1, Q_2)$ is increasing in product quantities $Q_1 + Q_2$ and Q_2. Therefore, $R_V^*(\hat{Q}_1, \hat{Q}_2) \geq R_V^*(Q_1, Q_2)$. The production cost $C(\hat{\mathbf{q}}) = \sum_{t \in \{A, B\}} \bar{c}_t \hat{q}_t = C(\bar{\mathbf{q}}) = \sum_{t \in \{A, B\}} \bar{c}_t \bar{q}_t$ because $C(\hat{\mathbf{q}}) - C(\bar{\mathbf{q}}) = (\bar{c}_i \cdot \bar{c}_j/\bar{c}_i - \bar{c}_j)\bar{q}_j = 0$. Therefore the profit for $\hat{\mathbf{q}}$ is at least as large as the profit for $\bar{\mathbf{q}}$. Therefore any arbitrary feasible production vector $\bar{\mathbf{q}}$ with $\bar{q}_j > 0$ is (weakly) dominated (in the sense of a (weakly) higher profit) by some feasible production vector $\hat{\mathbf{q}}$ with $\bar{q}_j = 0$. Therefore, there exists an optimal production vector with $\bar{q}_j = 0$. $\square$

Proof of Theorem 1. (Vertical Two Technologies). Applying (6) to the two-technology case, the firm's problem is to maximize the profit $\Pi(\bar{q}_A, \bar{q}_B) = R_V^*((1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B, \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B) - \bar{c}_A \bar{q}_A - \bar{c}_B \bar{q}_B$, where $R_V^*(\cdot, \cdot)$ is given by (5). Using (5), we can write $\Pi(\bar{q}_A, \bar{q}_B) = r_1(\bar{q}_A, \bar{q}_B) + r_2(\bar{q}_A, \bar{q}_B) - \bar{c}_A \bar{q}_A - \bar{c}_B \bar{q}_B$, where

$$r_1(\bar{q}_A, \bar{q}_B) = \begin{cases} x_1\left(1 - \dfrac{\bar{q}_A + \bar{q}_B}{S}\right)(\bar{q}_A + \bar{q}_B), & \bar{q}_A + \bar{q}_B \leq S/2 \\ \dfrac{x_1 S}{4}, & \bar{q}_A + \bar{q}_B > S/2 \end{cases}$$

$$r_2(\bar{q}_A, \bar{q}_B) = \begin{cases} (x_2 - x_1)\left(1 - \dfrac{\alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B}{S}\right)(\alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B), \\ \qquad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B \leq S/2 \\ \dfrac{(x_2 - x_1)S}{4}, \quad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B > S/2. \end{cases}$$

The firm's problem is to maximize $\Pi(\bar{q}_A, \bar{q}_B)$ subject to $\bar{q}_A \geq 0$ and $\bar{q}_B \geq 0$.

(i) Technologies have identical splits, i.e., $\alpha_{A2} = \alpha_{B2}$. By Definition 1, Technology A dominates Technology B if $c_A/y_A \leq c_B/y_B$, and vice versa (because the product-2 splits are identical). The proposition statement then follows directly from Lemma 1.

(ii) Technologies differ in their splits, i.e., $\alpha_{A2} > \alpha_{B2}$ because w.l.o.g. the technologies are labelled such that $\alpha_{A2} \geq \alpha_{B2}$. By Lemma 1, technology $j = \{A, B\}$ is not activated at optimality if it is dominated by technology $i = \{A, B\} \backslash j$, i.e., if $\bar{c}_i \leq \bar{c}_j$ and $\bar{c}_i/\alpha_{i2} \leq \bar{c}_j/\alpha_{j2}$, with at least one inequality being strict (see Definition 1 in the main body of the paper). If technology $j \in \{A, B\}$ is dominated by technology $i = \{A, B\} \backslash j$ then it is optimal to activate i only and the optimal quantity is given by Proposition 1. It then remains to consider the case in which neither technology dominates the other. Note that

$$\frac{\partial r_1(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_A} = \frac{\partial r_1(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_B}$$
$$= \begin{cases} x_1\left(1 - \dfrac{2(\bar{q}_A + \bar{q}_B)}{S}\right), & \bar{q}_A + \bar{q}_B \leq S/2 \\ 0, & \bar{q}_A + \bar{q}_B > S/2 \end{cases}$$

and

$$\frac{\partial^2 r_1(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_A^2} = \frac{\partial^2 r_1(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_B^2} = \begin{cases} -(2/S)x_1 < 0, & \bar{q}_A + \bar{q}_B \leq S/2 \\ 0, & \bar{q}_A + \bar{q}_B > S/2. \end{cases}$$

In addition, note that $\partial^2 r_1(\bar{q}_A, \bar{q}_B)/\partial \bar{q}_A \partial \bar{q}_B = 0$. It follows that the Hessian matrix is negative definite and hence $r_1(\bar{q}_A, \bar{q}_B)$ is jointly concave in $\bar{q}_A$ and $\bar{q}_B$. Similarly, for $i = A, B$ we have

$$\frac{\partial r_2(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_i} = \begin{cases} (x_2 - x_1)\alpha_{i2}(1 - 2(\alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B)/S), \\ \qquad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B \leq S/2 \\ 0, \quad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B > S/2 \end{cases}$$

and

$$\frac{\partial^2 r_2(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_i^2} = \begin{cases} -(x_2 - x_1)2\alpha_{i2}^2/S < 0, & \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B \leq S/2 \\ 0, & \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B > S/2. \end{cases}$$

In addition, note that $\partial^2 r_2(\bar{q}_A, \bar{q}_B)/\partial \bar{q}_A \partial \bar{q}_B = -2(x_2 - x_1) \cdot \alpha_{A2}\alpha_{B2}/S$. It follows that the Hessian matrix is negative semidefinite and hence $r_2(\bar{q}_A, \bar{q}_B)$ is (weakly) jointly concave in $\bar{q}_A$ and $\bar{q}_B$. Because $\Pi(\bar{q}_A, \bar{q}_B)$ is a linear combination of $r_1(\bar{q}_A, \bar{q}_B)$ and $r_2(\bar{q}_A, \bar{q}_B)$, it then follows that $\Pi(\bar{q}_A, \bar{q}_B)$ is jointly concave in $\bar{q}_A$ and $\bar{q}_B$. The optimal activation strategy (and production quantities) can be found by application of the first-order conditions subject to the nonnegative boundary conditions. Detailed algebra omitted for reasons of space.

We note that the optimal quantities in either case (i) or (ii) are as follows. If $\bar{c}_B \leq \bar{c}_L$ ($\bar{c}_B \geq \bar{c}_H$) then activate B only (A only) and the optimal yield-adjusted production quantity is given by the single-technology Proposition 1 using technology B's (A's) cost and split parameters. If $\bar{c}_L < \bar{c}_B < \bar{c}_H$ then activate both A and B and the optimal yield-adjusted production quantities are given by $(q_A^*, q_B^*) = (\hat{q}_A, \hat{q}_B)$ if $\hat{q}_B \leq \bar{q}_B$ and $(q_A^*, q_B^*) = (\bar{q}_A, \bar{q}_B)$ otherwise, where

$$\hat{q}_A = \frac{\lambda_B \gamma_A - \mu \gamma_B}{\lambda_A \lambda_B - \mu^2}; \quad \hat{q}_B = \frac{\lambda_A \gamma_B - \mu \gamma_A}{\lambda_A \lambda_B - \mu^2}$$

$$\bar{q}_A = \frac{S}{2} \frac{1 - (\bar{c}_A/\alpha_{A2})/(x_2 - x_1) - \alpha_{B2}}{\alpha_{A2} - \alpha_{B2}};$$

$$\bar{q}_B = \frac{S}{2} \frac{\alpha_{A2} - (1 - (\bar{c}_A)/\alpha_{A2}/(x_2 - x_1))}{\alpha_{A2} - \alpha_{B2}}$$

and $\lambda_A = x_1 + (x_2 - x_1)\alpha_{A2}^2$, $\lambda_B = x_1 + (x_2 - x_1)\alpha_{B2}^2$, $\mu = x_1 + (x_2 - x_1)\alpha_{A2}\alpha_{B2}$, $\gamma_A = (S/2)(x_1 + (x_2 - x_1)\alpha_{A2} - \bar{c}_A)$, and $\gamma_B = (S/2)(x_1 + (x_2 - x_1)\alpha_{B2} - \bar{c}_B)$. $\quad\square$

Corollary 1 (Vertical Adoption and Usage). *There exist adoption and displacement thresholds,* $A(\bar{c}_I, y_I, \alpha_{I2}, \alpha_{N2}) \geq D(\bar{c}_I, y_I, \alpha_{I2}, \alpha_{N2})$, *such that Technology N displaces Technology I if* $(c_N + k_N)/y_N \leq D(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$; *Technology N is used with Technology I if* $D(\bar{c}_I, \alpha_{I2}, \alpha_{N2}) < (c_N + k_N)/y_N < A(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$; *and Technology N is not adopted if* $(c_N + k_N)/y_N \geq A(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$.

Proof of Corollary 1. (Vertical Adoption and Usage). (i) The firm will build a capacity quantity for Technology N that exactly equals its planned production quantity. Therefore, the effective processing cost for Technology N is $c_N + k_N$. Note that Technology N will be adopted iff it will be optimally activated in the firm's technology portfolio, i.e., in consideration of the incumbent Technology I. The necessary and sufficient condition for Technology N to be activated if the firm also has Technology I can be obtain by adapting Theorem 1, which gives the optimal activation strategy for two arbitrary technologies denoted as A and B with $\alpha_{A2} \geq \alpha_{B2}$ without loss of generality. We first consider the case where $\alpha_{N2} \leq \alpha_{I2}$, i.e., the incumbent Technology I takes the role of A and N the role of B. Using Theorem 1, B is activated iff $\bar{c}_B < \bar{c}_H$, where $\bar{c}_H$ is given by Equation (8) and is a function of $\bar{c}_A$, α_{A2} and α_{B2}. Noting that $\bar{c}_A = \bar{c}_I$, $\alpha_{A2} = \alpha_{I2}$, $\bar{c}_B = (c_N + k_N)/y_N$, and $\alpha_{B2} = \alpha_{N2}$, and rearranging terms in (8), we obtain that N is activated iff $(c_N + k_N)/y_N \leq A(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$, where

$$A(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$$
$$= \begin{cases} \left(\dfrac{\bar{c}_I}{\alpha_{I2}}\right)\alpha_{N2}, & \bar{c}_I \leq \bar{c}_{\text{sat},I}(\alpha_{I2}) \\[2mm] \dfrac{(x_2 - x_1)(\alpha_{I2}\bar{c}_I + x_1(1 - \alpha_{I2}))\alpha_{N2} + x_1(\bar{c}_I - \bar{c}_{\text{sat},I})}{x_1 + (x_2 - x_1)\alpha_{I2}^2}, \\[2mm] & \text{otherwise,} \end{cases} \quad (A.1)$$

where $\bar{c}_t = c_t/y_t$ for $t \in \{I, N\}$ and $\bar{c}_{\text{sat},I}(\alpha_{I2}) = (x_2 - x_1) \cdot (1 - \alpha_{I2})\alpha_{I2}$. We next consider the case where $\alpha_{N2} > \alpha_{I2}$, i.e., Technology I takes the role of B and N the role of A. Using Theorem 1, A is activated iff $\bar{c}_B > \bar{c}_L$, where $\bar{c}_L$ is given by Equation (7) and is a function of $\bar{c}_A$, α_{A2} and α_{B2}. Noting that $\bar{c}_A = (c_N + k_N)/y_N$, $\alpha_{A2} = \alpha_{N2}$, $\bar{c}_B = \bar{c}_I$ and $\alpha_{B2} = \alpha_{I2}$, and rearranging terms in (7), we obtain that N is activated iff $(c_N + k_N)/y_N \leq A(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$, where $A(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$ is again given by (A.1).

(ii) Theorem 1 gives the optimal activation strategy for two arbitrary technologies denoted as A and B with $\alpha_{A2} \geq \alpha_{B2}$ without loss of generality. We first consider the case where $\alpha_{N2} \leq \alpha_{I2}$, i.e., the current technology I takes the role of A and N takes the role of B. Using Theorem 1, (i) B displaces A, i.e., only B is activated, iff $\bar{c}_B \leq \bar{c}_L$, and (ii) B complements A, i.e., both are activated, iff $\bar{c}_L < \bar{c}_B < \bar{c}_H$, where $\bar{c}_L$ and $\bar{c}_H$ are given by (7) and (8), respectively. Noting that $\bar{c}_A = \bar{c}_I$, $\alpha_{A2} = \alpha_{I2}$, $\bar{c}_B = (c_N + k_N)/y_N$ and $\alpha_{B2} = \alpha_{N2}$, and rearranging terms in (7) and (8), we obtain that N is a displacing technology if $(c_N + k_N)/y_N \leq D(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$ and a complementing technology iff $D(\bar{c}_I, \alpha_{I2}, \alpha_{N2}) < (c_N + k_N)/y_N \leq A(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$, where

$$D(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$$
$$= \begin{cases} \left(\dfrac{\bar{c}_I}{\alpha_{I2}}\right)\alpha_{N2}, & \alpha_{N2} \leq 1 - \dfrac{\bar{c}_I/\alpha_{I2}}{x_2 - x_1} \\[2mm] \dfrac{(x_1 + (x_2 - x_1)\alpha_{N2}^2)\bar{c}_I + x_1(x_2 - x_1)(\alpha_{N2} - \alpha_{I2})(1 - \alpha_{N2})}{x_1 + (x_2 - x_1)\alpha_{I2}\alpha_{N2}}, \\[2mm] & \text{otherwise;} \end{cases}$$
$$(A.2)$$

and $A(\bar{c}_1, \alpha_{12}, \alpha_{22})$ by (A.1). Next, consider the case of $\alpha_{N2} \geq \alpha_{I2}$, i.e., the current technology I takes the role of B and the useful innovation N takes the role of A. Using Theorem 1, (i) A displaces B, i.e., only A is activated, iff $\bar{c}_B \geq \bar{c}_H$, and (ii) A complements B, i.e., both are activated, iff $\bar{c}_L < \bar{c}_B < \bar{c}_H$, where $\bar{c}_L$ and $\bar{c}_H$ are given by (7) and (8), respectively. Noting that $\bar{c}_A = (c_N + k_N)/y_N$, $\alpha_{A2} = \alpha_{N2}$, $\bar{c}_B = \bar{c}_I$ and $\alpha_{B2} = \alpha_{I2}$, and rearranging terms in (7) and (8), we again obtain that N is a displacing technology if $(c_N + k_N)/y_N < D(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$ and a complementing technology iff $D(\bar{c}_I, \alpha_{I2}, \alpha_{N2}) < (c_N + k_N)/y_N \leq A(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$. Using Lemma S1, technology N is superior to I iff $(c_N + k_N)/y_N < \bar{c}_{\Pi_V^*}(\alpha_{N2})$. Now, by definition, an inferior technology can never displace a superior technology, and so we have $D(\bar{c}_I, \alpha_{I2}, \alpha_{N2}) \leq A(\bar{c}_I, \alpha_{I2}, \alpha_{N2})$. This completes the proof.

Proof of Proposition 2. (Horizontal Single Technology). Applying (11) to the single-technology case, the firm's problem is to maximize the profit function $\Pi(\bar{q}) = R_H^*((1 - \alpha_2)\bar{q}, \alpha_2\bar{q})) - \bar{c}\bar{q}$, where $R_H^*(\cdot, \cdot)$ is given by (10). Using (10), we can rewrite $R_H^*((1 - \alpha_2)\bar{q}, \alpha_2\bar{q}))$ as $R_H^*(\bar{q}) = r_1(\bar{q}) + r_2(\bar{q})$, where

$$r_1(\bar{q}) = \begin{cases} x_1\left(1 - \dfrac{(1 - \alpha_2)\bar{q}}{S_1}\right)(1 - \alpha_2)\bar{q}, & (1 - \alpha_2)\bar{q} \leq S_1/2 \\[2mm] \dfrac{x_1 S_1}{4}, & (1 - \alpha_2)\bar{q} > S_1/2; \end{cases}$$

$$r_2(\bar{q}) = \begin{cases} x_2\left(1 - \dfrac{\alpha_2\bar{q}}{S_2}\right)\alpha_2\bar{q}, & \alpha_2\bar{q} \leq S_2/2 \\[2mm] \dfrac{x_2 S_2}{4}, & \alpha_2\bar{q} > S_2/2. \end{cases}$$

The first and second derivatives of the profit function $\Pi(\bar{q}) = r_1(\bar{q}) + r_2(\bar{q}) - \bar{c}\bar{q}$ are given as follows.
   Case 1. $S_1/(1 - \alpha_2) \leq S_2/\alpha_2$.

$$\dfrac{d\Pi(\bar{q})}{d\bar{q}} = \begin{cases} x_1(1 - \alpha_2)\left(1 - \dfrac{2(1 - \alpha_2)\bar{q}}{S_1}\right) + x_2\alpha_2\left(1 - \dfrac{2\alpha_2\bar{q}}{S_2}\right) - \bar{c}, \\[2mm] \qquad\qquad\qquad \bar{q} \leq \dfrac{S_1}{2(1 - \alpha_2)} \\[3mm] x_2\alpha_2\left(1 - \dfrac{2\alpha_2\bar{q}}{S_2}\right) - \bar{c}, \quad \dfrac{S_1}{2(1 - \alpha_2)} < \bar{q} \leq \dfrac{S_2}{2\alpha_2} \\[3mm] -\bar{c}, \qquad\qquad\qquad \bar{q} > \dfrac{S_2}{2\alpha_2} \end{cases}$$

$$\dfrac{d^2\Pi(\bar{q})}{d\bar{q}^2} = \begin{cases} x_1(1 - \alpha_2)\left(-\dfrac{2(1 - \alpha_2)}{S_1}\right) + x_2\left(-\dfrac{2\alpha_2}{S_2}\right)\alpha_2, \\[2mm] \qquad\qquad\qquad \bar{q} \leq \dfrac{S_1}{2(1 - \alpha_2)} \\[3mm] x_2\left(-\dfrac{2\alpha_2}{S_2}\right)\alpha_2, \quad \dfrac{S_1}{2(1 - \alpha_2)} < \bar{q} \leq \dfrac{S_2}{2\alpha_2} \\[3mm] 0, \qquad\qquad\qquad \bar{q} > \dfrac{S_2}{2\alpha_2} \end{cases}$$

Case 2. $S_1/(1 - \alpha_2) > S_2/\alpha_2$.

$$\frac{d\Pi(\bar{q})}{d\bar{q}} = \begin{cases} x_1(1 - \alpha_2)\left(1 - \frac{2(1 - \alpha_2)\bar{q}}{S_1}\right) + x_2\alpha_2\left(1 - \frac{2\alpha_2\bar{q}}{S_2}\right) - \bar{c}, \\ \qquad\qquad\qquad \bar{q} \leq \frac{S_2}{2\alpha_2} \\ x_2\alpha_2\left(1 - \frac{2\alpha_2\bar{q}}{S_2}\right) - \bar{c}, \quad \frac{S_2}{2\alpha_2} < \bar{q} \leq \frac{S_1}{2(1 - \alpha_1)} \\ -\bar{c}, \qquad\qquad\qquad \bar{q} > \frac{S_1}{2(1 - \alpha_1)} \end{cases}$$

$$\frac{d^2\Pi(\bar{q})}{d\bar{q}^2} = \begin{cases} x_1(1 - \alpha_2)\left(-\frac{2(1 - \alpha_2)}{S_1}\right) + x_2\left(-\frac{2\alpha_2}{S_2}\right)\alpha_2, \\ \qquad\qquad\qquad \bar{q} \leq \frac{S_2}{2\alpha_2} \\ x_2\left(-\frac{2\alpha_2}{S_2}\right)\alpha_2, \quad \frac{S_2}{2\alpha_2} < \bar{q} \leq \frac{S_1}{2(1 - \alpha_1)} \\ 0, \qquad\qquad\qquad \bar{q} > \frac{S_1}{2(1 - \alpha_1)}. \end{cases}$$

It follows that in both cases $\Pi(\bar{q})$ is concave. Furthermore, it is strictly decreasing for $\bar{q} > S_2/(2\alpha_2)$ (in Case 1) and for $\bar{q} > S_1/(2(1 - \alpha_1))$ (in Case 2). Proof of proposition statement then follows by application of appropriate first-order condition and subsequent substitution into the profit function. □

Lemma 2 (Horizontal Coproducts Dominance). *For $i \in \{A, B\}$ let j denote the other technology, i.e., $j = \{A, B\}\backslash i$. If Technology i dominates Technology j then there exists an optimal production vector $\bar{\mathbf{q}}^*$ that does not use Technology j, i.e., $\bar{q}_j^* = 0$.*

Proof of Lemma 2. (Horizontal Coproducts Dominance). From the dominance Definition 2, Technology i dominates technology j iff $c_i/((1 - \alpha_{i2})y_i) \leq c_j/((1 - \alpha_{j2})y_j)$ and $c_i/(\alpha_{i2}y_i) \leq c_j/(\alpha_{j2}y_j)$; or equivalently Technology i dominates technology j = iff $\bar{c}_i/\alpha_{in} \leq \bar{c}_j/\alpha_{jn}$ for $n = 1, 2$ using the yield-adjusted cost notation and using $\alpha_{t1} = 1 - \alpha_{t2}$ for $t = i, j$.

Consider any arbitrary production vector $\bar{\mathbf{q}}$ with $\bar{q}_j > 0$. Construct a new production vector $\hat{\mathbf{q}}$ that is identical to $\bar{\mathbf{q}}$ except that $\hat{q}_j = 0$ and $\hat{q}_i = \bar{q}_i + (\alpha_{j\hat{n}}/\alpha_{i\hat{n}})\bar{q}_j$, where $\hat{n} = \arg\max_{n=1,2}\{\alpha_{jn}/\alpha_{in}\}$. The new quantity available in market $n = 1, 2$, i.e., $\hat{Q}_n$, is weakly larger than the original quantity Q_n because $Q_n = q_{n,i} + q_{n,j} = \alpha_{in}\bar{q}_i + \alpha_{jn}\bar{q}_j$ and (by the dominance definition) $\bar{c}_i/\alpha_{in} \leq \bar{c}_j/\alpha_{jn} \; \forall n \Rightarrow \alpha_{in}\bar{q}_i \geq \alpha_{jn}\bar{q}_j$, where the last inequality follows from the fact that $\hat{q}_i = \bar{q}_i + (\alpha_{j\hat{n}}/\alpha_{i\hat{n}})\bar{q}_j$ and recalling that $\hat{n}$ corresponds to the market segment with largest α_{jn}/α_{in} ratio. It then follows directly that $R_H^*(\hat{Q}_1, \hat{Q}_2) \geq R_H^*(Q_1, Q_2)$. The production cost $C(\hat{\mathbf{q}}) = \bar{c}_i\hat{q}_i = \bar{c}_j\hat{q}_j + \bar{c}_i(\alpha_{j\hat{n}}/\alpha_{i\hat{n}})q_j \leq \bar{c}_i\bar{q}_i + \bar{c}_j q_j = C(\bar{\mathbf{q}})$ because (by the dominance definition) $\bar{c}_i/\alpha_{in} \leq \bar{c}_j/\alpha_{jn}$ for $n = 1, 2$. Therefore the profit for $\hat{\mathbf{q}}$ is at least as large as the profit for $\bar{\mathbf{q}}$. Therefore any arbitrary feasible production vector $\bar{\mathbf{q}}$ with $\bar{q}_j > 0$ is (weakly) dominated (in the sense of a (weakly) higher profit) by some feasible production vector $\hat{\mathbf{q}}$ with $\bar{q}_j = 0$. Therefore, there exists an optimal production vector with $\bar{q}_j^* = 0$. □

Proof of Theorem 2. (Horizontal Two Technology). Applying (11) to the two-technology case, the firm's problem is to maximize the profit $\Pi(\bar{q}_A, \bar{q}_B) = R_H^*((1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B, \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B)) - \bar{c}_A\bar{q}_A - \bar{c}_B\bar{q}_B$, where $R_H^*(\cdot, \cdot)$ is given by (10).

Using (10), we can write $\Pi(\bar{q}_A, \bar{q}_B) = r_1(\bar{q}_A, \bar{q}_B) + r_2(\bar{q}_A, \bar{q}_B) - \bar{c}_A\bar{q}_A - \bar{c}_B\bar{q}_B$, where

$$r_1(\bar{q}_A, \bar{q}_B) = \begin{cases} x_1\left(1 - \frac{(1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B}{S_1}\right)((1 - \alpha_{A2})\bar{q}_A \\ \quad + (1 - \alpha_{B2})\bar{q}_B), \quad (1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B \leq S_1/2 \\ \frac{x_1 S_1}{4}, \qquad\qquad (1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B > S_1/2 \end{cases}$$

$$r_2(\bar{q}_A, \bar{q}_B) = \begin{cases} x_2\left(1 - \frac{\alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B}{S_2}\right)(\alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B), \\ \qquad\qquad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B \leq S_2/2 \\ \frac{x_2 S_2}{4}, \quad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B > S_2/2. \end{cases}$$

The firm's problem is to maximize $\Pi(\bar{q}_A, \bar{q}_B)$ subject to $\bar{q}_A \geq 0$ and $\bar{q}_B \geq 0$.

(i) Technologies have identical splits, i.e., $\alpha_{A2} = \alpha_{B2}$. By Definition 1, Technology A dominates Technology B if $c_A/y_A \leq c_B/y_B$, and vice versa (because the product-2 splits are identical). The proposition statement then follows directly from Lemma 2.

(ii) Technologies differ in their splits, i.e., $\alpha_{A2} > \alpha_{B2}$ because w.l.o.g. the technologies are labelled such that $\alpha_{A2} \geq \alpha_{B2}$. By Lemma 2, technology $j \in \{A, B\}$ is not activated at optimality if it is dominated by technology $i = \{A, B\}\backslash j$, i.e., if $\bar{c}_i/(1 - \alpha_{i2}) \leq \bar{c}_j/(1 - \alpha_{j2})$ and $\bar{c}_i/\alpha_{i2} \leq \bar{c}_j/\alpha_{j2}$, with at least one inequality being strict. If technology $j \in \{A, B\}$ is dominated by technology $i = \{A, B\}\backslash j$ then it is optimal to activate i only and the optimal quantity is given by Proposition 1. It then remains to consider the case in which neither technology dominates the other. Note that for $i \in \{A, B\}$ we have

$$\frac{\partial r_1(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_i} = \begin{cases} x_1(1 - \alpha_{i2})\left(1 - \frac{2((1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B)}{S_1}\right), \\ \qquad\qquad (1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B \leq S_1/2 \\ 0, \quad (1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B > S_1/2 \end{cases}$$

and

$$\frac{\partial^2 r_1(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_i^2} = \begin{cases} -\frac{2(1 - \alpha_{i2})^2}{S_1}x_1 < 0, \quad (1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B \leq S_1/2 \\ 0, \qquad\qquad (1 - \alpha_{A2})\bar{q}_A + (1 - \alpha_{B2})\bar{q}_B > S_1/2. \end{cases}$$

In addition, note that $\partial^2 r_1(\bar{q}_A, \bar{q}_B)/\partial \bar{q}_A \partial \bar{q}_B = -2x_1(1 - \alpha_{A2}) \cdot (1 - \alpha_{B2})/S_1 < 0$. It follows that the Hessian matrix is negative semidefinite and hence $r_1(\bar{q}_A, \bar{q}_B)$ is (weakly) jointly concave in $\bar{q}_A$ and $\bar{q}_B$. Similarly, for $i \in \{A, B\}$ we also have

$$\frac{\partial r_2(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_i} = \begin{cases} x_2\alpha_{i2}\left(1 - 2(\alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B)/S_2\right), \\ \qquad\qquad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B \leq S_2/2 \\ 0, \quad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B > S_2/2 \end{cases}$$

and

$$\frac{\partial^2 r_2(\bar{q}_A, \bar{q}_B)}{\partial \bar{q}_i^2} = \begin{cases} -x_2(2\alpha_{i2}^2/S_2) < 0, \quad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B \leq S_2/2 \\ 0, \qquad\qquad \alpha_{A2}\bar{q}_A + \alpha_{B2}\bar{q}_B > S_2/2. \end{cases}$$

In addition, note that $\partial^2 r_2(\bar{q}_A, \bar{q}_B)/\partial \bar{q}_A \partial \bar{q}_B = -2x_2\alpha_{A2}\alpha_{B2}/S_2$. It follows that the Hessian matrix is negative semidefinite and hence $r_2(\bar{q}_A, \bar{q}_B)$ is (weakly) jointly concave in $\bar{q}_A$

and $\bar{q}_B$. Because $\Pi(\bar{q}_A, \bar{q}_B)$ is a linear combination of $r_1(\bar{q}_A, \bar{q}_B)$ and $r_2(\bar{q}_A, \bar{q}_B)$, it then follows that $\Pi(\bar{q}_A, \bar{q}_B)$ is jointly concave in $\bar{q}_A$ and $\bar{q}_B$. The optimal activation strategy (and production quantities) can be found by application of the first-order conditions subject to the nonnegative boundary conditions. Detailed algebra omitted for reasons of space. In what follows we present the optimal production quantities when dual activation is optimal, i.e., when $\bar{c}_L < c_B/y_B < \bar{c}_H$ in part (ii) of the theorem. In this case, optimal yield-adjusted production quantities are given by $(q_A^*, q_B^*) = (\hat{q}_A, \hat{q}_B)$ if $\hat{q}_B \le \bar{q}_B$ and $\hat{q}_A \le \bar{\bar{q}}_A$, by $(q_A^*, q_B^*) = (\bar{q}_A, \bar{q}_B)$ if $\hat{q}_B > \bar{q}_B$, and by $(q_A^*, q_B^*) = (\bar{\bar{q}}_A, \bar{\bar{q}}_B)$ otherwise, where

$$\hat{q}_A = \frac{\lambda_B \gamma_A - \mu \gamma_B}{\lambda_A \lambda_B - \mu^2}, \quad \hat{q}_B = \frac{\lambda_A \gamma_B - \mu \gamma_A}{\lambda_A \lambda_B - \mu^2},$$

$$\bar{q}_A = \frac{S}{2} \frac{\alpha_{B2}}{\alpha_{A2}(1-\alpha_{B2}) - (1-\alpha_{A2})\alpha_{B2}}$$
$$\cdot \left(\hat{s}_2 \left(1 - \frac{\bar{c}_A/\alpha_{A2}}{x_2} \right) - (1-\hat{s}_2) \right),$$

$$\bar{q}_B = \frac{S}{2} \frac{(1-\hat{s}_2)\alpha_{A2} - \hat{s}_2(1-\alpha_{A2})(1-(\bar{c}_A/\alpha_{A2})/x_2)}{\alpha_{A2}(1-\alpha_{B2}) - (1-\alpha_{A2})\alpha_{B2}},$$

$$\bar{\bar{q}}_A = \frac{S}{2} \frac{\hat{s}_2(1-\alpha_{B2}) - (1-\hat{s}_2)\alpha_{B2}(1-(\bar{c}_A/(1-\alpha_{A2}))/x_1)}{\alpha_{A2}(1-\alpha_{B2}) - (1-\alpha_{A2})\alpha_{B2}},$$

$$\bar{\bar{q}}_B = \frac{S}{2} \frac{1-\alpha_{A2}}{\alpha_{A2}(1-\alpha_{B2}) - (1-\alpha_{A2})\alpha_{B2}}$$
$$\cdot \left((1-\hat{s}_2) \left(1 - \frac{\bar{c}_A/(1-\alpha_{A2})}{x_1} \right) - \hat{s}_2 \right),$$

and

$$\lambda_A = \frac{x_1(1-\alpha_{A2})^2}{1-\hat{s}_2} + \frac{x_2\alpha_{A2}^2}{\hat{s}_2}, \quad \lambda_B = \frac{x_1(1-\alpha_{B2})^2}{1-\hat{s}_2} + \frac{x_2\alpha_{B2}^2}{\hat{s}_2},$$

$$\mu = \frac{x_1(1-\alpha_{A2})(1-\alpha_{B2})}{1-\hat{s}_2} + \frac{x_2\alpha_{A2}\alpha_{B2}}{\hat{s}_2},$$

$$\gamma_A = \frac{S}{2}(x_1 + (x_2-x_1)\alpha_{A2} - \bar{c}_A), \quad \gamma_B = \frac{S}{2}(x_1 + (x_2-x_1)\alpha_{B2} - \bar{c}_B).$$

Recall that the actual production quantities are obtained by the conversion $q_t = \bar{q}_t/y_t$. □

Proof of Proposition 3. (Ideal Market Product-2 Split). Recall that $\bar{c} = c/y$. Let $\Pi_H^*(\bar{c}, \alpha_2)$ denote the optimal profit for a technology with yield-adjusted cost $\bar{c} > 0$ and split α_2. $\Pi_H^*(\bar{c}, \alpha_2)$ is given in Proposition 2 and is a continuous and differentiable function of α_2. Differentiating with respect to α_2, we obtain

$$\frac{d\Pi_H^*}{d\alpha_2} = \frac{S}{2}\hat{s}_2 x_2 \left(1 - \frac{\bar{c}}{\alpha_2 x_2} \right) \frac{\bar{c}}{\alpha_2^2 x_2},$$
$$0 \le \alpha_2 < \hat{s}_2 \text{ and } \frac{\bar{c}}{\alpha_2 x_2} \le 1 - \frac{\alpha_2}{1-\alpha_2}\frac{1-\hat{s}_2}{\hat{s}_2}$$

$$\frac{d\Pi_H^*}{d\alpha_2} = -\frac{S}{2}(1-\hat{s}_2)x_1 \left(1 - \frac{\bar{c}}{(1-\alpha_2)x_1} \right) \frac{\bar{c}}{(1-\alpha_2)^2 x_1},$$
$$\hat{s}_2 < \alpha_2 \le 1 \text{ and } \frac{\bar{c}}{(1-\alpha_2)x_1} \le 1 - \frac{1-\alpha_2}{\alpha_2}\frac{\hat{s}_2}{1-\hat{s}_2}$$

$$\frac{d\Pi_H^*}{d\alpha_2} = \frac{S}{2} \frac{x_1 + (x_2-x_1)\alpha_2 - \bar{c}}{x_1\hat{s}_2(1-\alpha_2)^2 + x_2(1-\hat{s}_2)\alpha_2^2} \Big[x_1 x_2(\hat{s}_2 - \alpha_2)$$
$$+ \bar{c}(x_2(1-\hat{s}_2)\alpha_2 - x_1\hat{s}_2(1-\alpha_2)) \Big], \quad \text{otherwise.}$$

Recall that $\bar{c} < x_1 + (x_2-x_1)\alpha_2$ by assumption; else production and profit are both zero. Now, (i) $d\Pi_H^*/d\alpha_2 > 0$ if $0 \le \alpha_2 < \hat{s}_2$

and $\bar{c}/(\alpha_2 x_2) \le 1 - (\alpha_2/(1-\alpha_2))((1-\hat{s}_2)/\hat{s}_2)$; (ii) $d\Pi_H^*/d\alpha_2 < 0$ if $\hat{s}_2 < \alpha_2 \le 1$ and $\bar{c}/((1-\alpha_2)x_1) \le 1 - ((1-\alpha_2)/\alpha_2)(\hat{s}_2/(1-\hat{s}_2))$, and otherwise (iii) $d\Pi_H^*/d\alpha_2 = h(\alpha_2)f(\alpha_2)$ where $h(\alpha_2) > 0$ and $f(\alpha_2)$ is linear. First, consider the case of $\bar{c} \ge x_1$. We then have $\bar{c}/((1-\alpha_2)x_1) > 1 - ((1-\alpha_2)/\alpha_2)(\hat{s}_2/(1-\hat{s}_2))$ and, in addition, $f(\alpha_2) > 0$ for all $0 \le \alpha_2 \le 1$. Therefore, $d\Pi_H^*/d\alpha_2 > 0$ for all $0 \le \alpha_2 \le 1$ and $\alpha_2^*(\bar{c}) = 1$. Next consider the case of $\bar{c} < x_1$. In this case, $f(\alpha_2) > 0$ is linear decreasing in α_2 and $f(\alpha_2)|_{\alpha_2=\hat{s}_2} > 0$ and $f(\alpha_2)|_{\alpha_2=1} < 0$. Therefore, $d\Pi_H^*/d\alpha_2 > 0$ crosses zero (from the positive side) exactly once, and thus $\Pi_H^*(\bar{c}, \alpha_2)$ is a quasiconcave function. It follows that $\alpha_2^*(\bar{c})$ is given by the solution to the first-order condition. This is obtained by solving for $h(\alpha_2) = 0$, which yields $\alpha_2^*(\bar{c}) = x_1\hat{s}_2(x_2 - \bar{c})/(x_2(1-\hat{s}_2)(x_1 - \bar{c}) + x_1\hat{s}_2(x_2 - \bar{c}))$. This completes the proof.

Corollary 2 (Horizontal Adoption and Usage Under Identical (Effective) Costs). *Let Technology I* (incumbent) *and Technology N* (new) *have the same* (effective) *yield-adjusted costs, i.e., $c_I/y_I = (c_N + k_N)/y_N$.*

(i) If $\alpha_{I2} < \alpha_2^*$ then Technology N is adopted iff $\alpha_{N2} > \alpha_{I2}$. Furthermore, Technology N displaces Technology I if $\alpha_{I2} < \alpha_{N2} \le \alpha_2^*$ but Technology N is used together with Technology I if $\alpha_{N2} > \alpha_2^*$.

(ii) If $\alpha_{I2} > \alpha_2^*$ then Technology N is adopted iff $\alpha_{N2} < \alpha_{I2}$. Furthermore, Technology N displaces Technology I if $\alpha_2^* \le \alpha_{N2} < \alpha_{I2}$ but Technology N is used together with Technology I if $\alpha_{N2} < \alpha_2^*$.

Proof of Corollary 2. (Horizontal Adoption and Usage Under Identical Cost). If the new technology is adopted, the firm will build a capacity quantity for Technology N that exactly equals its planned processing quantity. Therefore, the effective yield-adjusted cost for Technology N is $(c_N + k_N)/y_N$. By assumption, $c_I/y_I = (c_N + k_N)/y_N$. Define $\bar{c} = c_I/y_I = (c_N + k_N)/y_N$. Let $\Pi(\bar{q}_I, \bar{q}_N)$ denote the profit if the firm processes $\bar{q}_I$ units on technology I and $\bar{q}_N$ units on technology N. Because I and N have equal processing costs, i.e., $\bar{c}_I = \bar{c}_N = \bar{c}$, the same profit $\Pi(\bar{q}_I, \bar{q}_N)$ can be obtained by processing $\bar{q}_k = \bar{q}_N + \bar{q}_I$ units on a (hypothetical) single technology k with the same yield-adjusted cost $\bar{c}$ but a split of $\alpha_{k2} = \gamma\alpha_{I2} + (1-\gamma)\alpha_{N2}$, where $\gamma = \bar{q}_I/(\bar{q}_I + \bar{q}_N)$. From Proposition 3, there exists an "ideal" split α_2^* for any technology with yield-adjusted cost $\bar{c}$ such that the optimal profit is highest at α_2^*. If α_{I2} and α_{N2} lie on opposite sides of α_2^* then there always exists a blending fraction γ such that the two technologies are used in proportions that are equivalent to a single technology with split of α_2^*. If α_{I2} and α_{N2} lie on the same side of α_2^* then the firm cannot attain the ideal split and, moreover, should only use the technology whose split lies closest to α_2^*. This latter point follows from the fact (established in proof of Proposition 3) that the optimal profit (for a given yield-adjusted cost $\bar{c}$) is a quasiconcave function of the split α_2. Proof of corollary statements (i) and (ii) then follow. □

Endnotes

[1] Devices produced on 300 mm wafer have been estimated to be approximately 30% cheaper than on 200 mm wafer. Future increases in wafer size (e.g., from 300 mm to 450 mm) are predicted to deliver significantly smaller savings because the lithography cost scales in the wafer size and this cost will account for a large portion of the cost per device.

[2] Our discussion focuses on the theoretical operations literature, but we note that there is a long-established and extensive practice-focused, decision-support literature grounded in the petrochemical

industry, e.g., Bodington and Baker (1990) and references therein, and the semiconductor industry, e.g., Leachman et al. (1996) and Denton et al. (2006), and these papers reflect coproduct considerations in places.

[3] Intel, for example, concurrently produced Pentium 4 chips (that were graded by speed) on both 200 mm and 300 mm wafer technologies (Millman 2002).

[4] When the base unit refers to the weight of some key input, e.g., tonne of salt in the chloralkali process, then production quantities can naturally take on any nonnegative real number. If the base unit refers to some physical entity, e.g., semiconductor wafer start, we assume the base unit is very small relative to overall production so that integer considerations can be ignored.

[5] When dual technology activation is optimal, i.e., $\bar{c}_L < c_B/y_B < \bar{c}_H$, we present the optimal production quantities at the end of the proof. When single technology activation is optimal in (i) or (ii), the optimal production quantity is given by Proposition 1 using the activated technology's parameters.

[6] Consider an incumbent technology defined by the following yield-adjusted cost and product-2 split $(c_I/y_I, \alpha_{I2}) = (0.205, 0.15)$ and a new technology defined by $((c_N + k_N)/y_N, \alpha_{N2}) = (1.15, 0.95)$, i.e., significantly higher effective cost and split. The new technology delivers a 9% lower stand-alone profit than the incumbent but it increases the firm's profit by 7% if adopted and used with the incumbent. As another example, a new technology $((c_N + k_N)/y_N, \alpha_{N2}) = (0.16, 0.1)$ has a stand-alone profit that is 10% lower than an incumbent technology $(c_I/y_I, \alpha_{I2}) = (0.987, 0.85)$ but the new technology increases the firm's profit by 9% if adopted and used with the incumbent.

[7] When dual technology activation is optimal, i.e., $\bar{c}_L < c_B/y_B < \bar{c}_H$, we present the optimal production quantities at the end of the proof. When single technology activation is optimal in (i) or (ii), the optimal production quantity is given by Proposition 2 using the activated technology's parameters.

[8] For example, consider an incumbent technology defined by the following yield-adjusted cost and product-2 split $(c_I/y_I, \alpha_{I2}) = (0.262, 0.15)$ and a new technology defined by $((c_N + k_N)/y_N, \alpha_{N2}) = (1.0, 0.95)$. The new technology delivers a 34% lower stand-alone profit than the incumbent but it increases the firm's profit by 20% if adopted and used with the incumbent.

[9] Figure 5 is obtained by setting $x_1 = 1.0$, $x_2 = 3.5$, $\alpha_{A2} = 0.8$, $\alpha_{B2} = 0.2$. The market size $\tilde{S}$ has a uniform distribution with a mean of 1 and a variance of 1/6.

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