Processing Time Ambiguity and Port Competitiveness

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Container seaports play an important role in cross-border logistics as firms increasingly expand their global footprint in sourcing, manufacturing and distribution. Besides convenience of access to hinterland regions, a key metric for a port's attractiveness is its processing time, that is, its ability to clear goods within a consistent, predictable time frame. Due to differences in infrastructure, government regulations, and operating procedures, ports may exhibit different degrees of predictability in processing times: some are more predictable while others are more ambiguous. We study how ambiguity in processing times affects a port's attractiveness under various circumstances. We find that even if a port maintains a consistent expected processing time, increased ambiguity can still affect its attractiveness to firms, although not always negatively. The effect of ambiguity depends on its nature, whether the shipments are time-sensitive, attitudes toward ambiguity, and trade terms surrounding shipments.

1. Introduction

Managing transit lead times is a critical task in global supply chain management. This is especially true for perishable products, which include frozen foods requiring temperature-controlled containers, and products with short selling seasons or life cycles, such as Christmas decorations and customized ASICs (application-specific integrated circuits). Because perishable products have a limited selling window, uncertainties around transit time can have serious consequences for profitability.

Based on a survey of 60 international carriers, Chung and Chiang (2011) report that only a handful achieve a transit schedule reliability of 90% or greater. Uncertainties in transit time can be categorized into several stages: hinterland to port (ocean, land, or air), port processing (export clearance), and port to destination (which includes ocean/land/air shipping, import clearance, and transportation to the hinterland). Many factors can influence transit time, some of which are external, such as weather, congestion, and road conditions, which affect all shipments, while others are operator-related, such as capacity, fleet network, and scheduling. The impact of these factors on transit lead time, however, is often predictable: for example, firms can check weather forecasts and examine carrier schedules to make reasonable estimates.

What is unpredictable and yet significantly influences transit time is processing efficiency in terms of customs clearance. Grosso and Monteiro (2011) report that customs procedures and characteristics, such as hours and efficiency, are among the most important factors in port selection (pp. 152–154). Similarly, Sanchez et al. (2011) and Sayareh and Alizmini (2014) note that time efficiency and delays are key contributing factors to a port's attractiveness (pp. 155; 89). Tiwari et al. (2003) find that congestion negatively affects port selection, while efficiency has a positive impact.[1] Because of complex regulations and safety mandates, the time required for shipments to clear customs is highly unpredictable. Hitachi, for example, faces 4000 pages of regulations when shipping components to the United States, and "even if only one component has been misclassified, the entire shipment is likely to be held up in customs until that item is in compliance" (Gary 2005). Similar observations have been made on exporting procedures, where government regulations and port bureaucracy can significantly influence port processing times (Persson et al.

2009). Furthermore, such complex processing procedures can cause shipments to miss their vessels, leading to further delays down the line (Veenstra 2014).

Based on a dataset of 44,723 container shipments from Asia to the United States for a large retailer in 2011, we find an average time at sea of 17.07 days (coefficient of variation 0.41) and average time at port (from ship arrival to out gate) of 7.50 days (coefficient of variation 0.71).[2] This suggests that the average time spent at port can be both very significant (around 44% of the length of the sea journey itself) and highly variable (coefficient of variation 0.71). Besides physical location and access to hinterland (Cheon et al. 2017), therefore, a port's attractiveness to a shipper can be significantly influenced by its efficiency and predictability in customs clearance and other procedures related to the clearing of a shipment. In this study, we study the impact of processing time predictability and physical location on a port's attractiveness to shippers and forwarders. For the sake of clarity, we focus our discussion on maritime ports of origin, but our analysis applies equally to land crossings or airports that handle either import or export processes.

As an illustration, consider the ports of Shenzhen and Hong Kong. Both are deep water ports capable of handling the largest container ships, but in terms of physical location, the Shenzhen port is closer to and thus offers more convenient access to the hinterland of the Pearl River Delta (PRD) than the Hong Kong port does. However, the Hong Kong port is often considered more efficient with more predictable processing times (Hong Kong Trade Development Council 2013).[3]

Predictability refers to a port's ability to consistently adhere to predesignated processing time standards. As processing times are inherently uncertain, predictability does not imply the processing times is deterministic but rather means that it has a consistently known distribution. In contrast, processing times are unpredictable when there is a set of possible distributions. Such unpredictability can be caused by many factors, such as opaque inspection procedures, paucity of information, and complex, caprice government regulations. Firms can combine these factors with estimation techniques such as market research, expert opinions, and looks-like analysis to generate a range of possible distributions of a port's processing time.[4] We define these possible distributions collectively as an *ambiguity set*. Because the ambiguity set contains beliefs (estimates) about possible processing time distributions, it will expand, that is, include a wider range of possible distributions, with decreased predictability but will shrink with increased predictability. Processing times do not exhibit ambiguity (but still exhibit uncertainty) when the ambiguity set contains only one distribution.

Isolating ambiguity from processing time uncertainty (e.g., as captured by a known distribution) offers several advantages. Firstly, ambiguity set explicitly captures a firm's uncertainty about the relevance of a particular distribution as the "correct" one. Secondly, it allows multiple distributions to be incorporated into a firm's forecasting and decision-making process. Thirdly, doing so allows the impact of ambiguity to be contrasted with that of uncertainty over a firm's preference for a port, particularly when the firm is ambiguity-averse. As such, although uncertainty over processing times exists in both cases (predictable or ambiguous), the ramifications for the shipper can vary substantially. A key question we explore is how the processing time ambiguity of a port affects its attractiveness to shippers. It is of significant managerial interest to understand whether a reduction in processing time ambiguity can enhance a port's competitiveness despite a locational disadvantage.[5]

A port's attractiveness may also be influenced by the International Commercial Terms (Incoterms), e.g., whether the trade terms are free on board (FOB) or cost insurance and freight (CIF). According to Lee and Wong (2007), the shipper (supplier) typically decides on the port of export, and hence attractive ports to buyers may not necessarily appeal to suppliers. We therefore consider both suppliers' and buyers' preferences and explore how ambiguity interacts with Incoterms to affect a port's attractiveness.

2. Related Literature

Ambiguity in lead time is related to the uncertain supply literature, which can be roughly classified into three different but related streams: supply disruption, random yield, and stochastic lead time. This research is most closely aligned with the third stream.

The problem of stochastic lead time is typically studied in multi-period inventory models as opposed to newsvendor models. Bagchi et al. (1986), Song (1994), and Chopra et al. (2008) study the effect of lead time variability on service levels as well as the expected cost for a single-item inventory system. Humair et al. (2013) study an inventory model that incorporates stochastic lead times with a guaranteed service level. Chopra et al. (2008) point out that the commonly-used normal approximation of lead time distribution can lead to erroneous safety stock levels. They examine several individual distributions independently, and thus the issue of ambiguity is irrelevant. Other related works, such as Chu et al. (1994), Fujiwara and Sedarage (1997) and Proth et al. (1997), investigate the effect of stochastic lead times on expected cost in multi-item inventory models. Zipkin (2000) provides a more detailed treatment of stochastic lead times in multi-period inventory models.

However, none of the above research examines the effect of ambiguity on the attractiveness of supply sources such as ports.

Specific to the ocean shipping industry, Vernimmen et al. (2007) find significant uncertainty in ocean shipping transit times, which has serious cost implications on shippers' and consignees' bottom lines. Djankov et al. (2010) further quantify the effect of ocean shipping delays on the volume of international trade. From a modeling perspective, Meng et al. (2013) broadly review the literature on container routing and scheduling which have significant implications for ocean shipping lead times. While we focus on the ocean shipping industry, our study is also applicable (albeit with non-trivial differences) to the air transport industry, where transit time uncertainty also has a significant impact on the attractiveness of competing airports and carriers (see, e.g., Hao and Hansen 2014, Koppelman et al. 2008, Koster et al. 2011).

While the above stream of research treats stochastic lead times as given, another stream of literature investigates the active management or estimation of stochastic lead times to improve supply chain performance. Notteboom (2006) points out the importance of understanding the root cause of variability in ocean transit time and suggests methods of improving schedule stability. From a modeling perspective, Brooks et al. (2016) propose a Bayesian hierarchical model that allows a more refined estimation of ocean shipping transit times via different ports in close proximity. Beyond the ocean shipping industry, Chaharsooghi and Heydari (2010) study the relative benefits of reducing average lead time or variability in lead time, and find through simulation that variability reduction has a larger impact on supply chain performance.

Our work is related to the stream of literature distinguishing ambiguity from uncertainty. Camerer and Weber (1992) summarize theoretical development in modeling uncertainty and ambiguity in the economics literature, while Schrader et al. (1993) illustrate that ambiguity can influence choices of solution differently from uncertainty. Dequech (2000) distinguishes uncertainty from fundamental ambiguity, which refers to situations where further information to increase the reliability of future estimates cannot be obtained through waiting. In such scenarios, it is neither useful nor relevant to investigate the commitment timing of buyers' purchasing decisions. Recently, Klibanoff et al. (2005) explore decision-making under ambiguity and show that ambiguity and uncertainty can be separated such that a decision-maker may exhibit different attitudes toward ambiguity and risk.

Our work is also closely related to the literature on factors that influence a port's attractiveness. Malchow and Kanafani (2004) suggest that the most significant factor influencing attractiveness is physical location. In China, however, Tiwari et al. (2003) note that port congestion and fleet size of shipping lines also play important roles. From an organizational perspective, Cheon et al. (2010) show that a port's ownership structure and institutional reforms can improve its efficiency and total productivity. Our research complements the above research in that ports cannot change their physical location and other factors enhancing their attractiveness must therefore be considered.

In closing, we note that while there is extensive literature on stochastic lead times and inventory policies, no paper to date has explicitly linked lead time ambiguity to firms' preferences in the context of port literature to the best of our knowledge. A key contribution of this research is to provide one such link. The rest of the study is organized as follows: we describe the model in section 3 and conduct a preliminary analysis in section 4. The effect of ambiguity is studied in section 5 and we complement this analysis with a numerical study in section 6. We conclude the study in section 7. All proofs are provided in the online supplement.
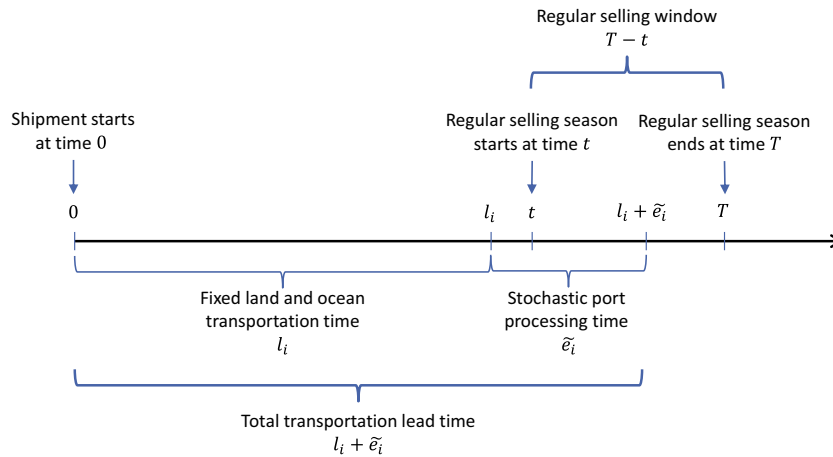
3. Model

Consider a buyer (consignee) that purchases a certain amount of a perishable product from a foreign supplier (shipper). The product has a short shelf life, such as fresh/cold produce, or a short selling season, such as seasonal or high-tech products, and is hence sensitive to transit lead times. The lead time is composed of several components: transportation from the supplier hinterland to an ocean or land port, port processing (e.g., customs inspection), ocean shipping and import processing, and finally transportation to the buyer's local market.

To focus on the effect of ambiguity in port processing time, we single out the processing time of port i as $\tilde{e}_i$ and aggregate all other components of the transit lead time from the hinterland to the buyer's market via port i as l_i. The total transit lead time via port i is therefore $l_i + \tilde{e}_i$. To reveal the effect of $\tilde{e}_i$ on port attractiveness, we further assume that l_i is fairly predictable and hence deterministic. Note that the case of stochastic l_i can be approximated in our model by maintaining the expected l_i and aggregating the stochastic portion into $\tilde{e}_i$.

Within a regular selling window from time t to T, the product commands the full market price of p_f but once this window is exceeded the product can only be sold at a discounted price $p_d < p_f$. For notational convenience, we adopt the convention that the buyer places the order at time 0 and that the supplier's production time is instantaneous, such that the order arrives by time $l_i + \tilde{e}_i$ when the product transits through port i. If the shipment arrives before time t,

Figure 1 Transportation Lead Time and the Time Window of the Regular Selling Season [Color figure can be viewed at wileyonlinelibrary.com]

the buyer incurs a unit holding cost of h per unit time. Figure 1 illustrates the relationship between transportation lead time ($\tilde{e}_i + l_i$) and the regular selling time window ($T - t$).

Market demand D is random and follows a general distribution $D \sim G(\cdot)$. The buyer places an order at time 0 with an order size of Q to meet market demand. If the shipment arrives within the regular selling window, the system behaves similarly to a news vendor problem: unmet demand is lost and the leftover inventory is salvaged at price p_d. On the other hand, if the shipment misses the regular selling window, then the product can only be sold at the discounted price p_d.

3.1. Processing Time Ambiguity
We refer to port i as having a predictable processing time if $\tilde{e}_i$ follows a *known* distribution, that is, $\tilde{e}_i \sim F_i(\cdot)$. In contrast, port i's processing time is ambiguous if there is a set of *plausible* distributions $\mathcal{F}_i = \{F_{i,1}(\cdot), F_{i,2}(\cdot), \ldots, F_{i,M_i}(\cdot)\}$, and the firm is unsure which distribution is 'correct' for $\tilde{e}_i$. The firm's subjective assessment of the likelihood that $\tilde{e}_i$ is drawn from $F_{i,k}(\cdot)$, $k \in \{1, \ldots, M\}$ is $\rho_{i,k} \geq 0$, where $\rho_{i,k}$ is the second-order probability over the first-order distribution $F_{i,k}$. The processing time at port i becomes more ambiguous as the second-order probability becomes more diffuse, that is, $\max_{k \neq l} |\rho_{i,k} - \rho_{i,l}| \to 0$, and becomes more predictable as $\max_{k \neq l} |\rho_{i,k} - \rho_{i,l}| \to 1$. As such, a predictable processing time is a special case of ambiguous processing time with $\rho_{i,k} = 1$ and $\rho_{i,l \neq k} = 0$ for some k. Segal (1987), Klibanoff et al. (2005), and Saghafian and Tomlin (2016) provide further examples, and discuss the properties and applications of ambiguity represented by second-order probability.

It is worth pointing out that if the firm is ambiguity-neutral then an ambiguous processing time is mathematically equivalent to a predictable processing time defined by $\bar{F}_i(\cdot) \sim \Sigma_k \rho_{i,k} F_{i,k}(\cdot)$. Otherwise if the firm is ambiguity-averse (or ambiguity-loving) then this equivalence breaks down (Proposition 1, Klibanoff et al. 2005). Recall that, even if the firm is ambiguity-neutral, separating predictable and ambiguous processing times is conceptually useful (see section 1).

Suppose, for example, port 1 has a predictable processing time with a known distribution $F_1(\cdot)$, whereas port 2 has an ambiguous processing time with two plausible distributions $F_{2,1}(\cdot)$ and $F_{2,2}(\cdot)$. If port 1 is selected, the firm knows that the shipment will clear the port in x days with probability $F_1(x)$. In contrast, if port 2 is selected, this probability is uncertain: it can be either $F_{2,1}(x)$ or $F_{2,2}(x)$. In this case, the firm relies on a subjective assessment (i.e., second-order probability) of whether $F_{2,1}$ or $F_{2,2}$ is more likely to be 'correct' to estimate the probability that the shipment will clear the port in x days. Thus, in the former case the probability of clearing in x days is deterministic, whereas in the latter case it is conceptually probabilistic. Modeling the latter case with second-order probability is therefore conceptually more appealing if the processing time is influenced by several distinct random factors (e.g., congestion, inspection, demurrage and detention) and the firm is unsure which factors are active.[6]

Increased ambiguity may not necessarily lead to increased (overall) variance. Suppose port i's processing time is governed by $\mathcal{F}_i = \{F_{i,k}\}$, $k \in \{1, 2, 3\}$. A predictable processing time can be constructed by setting the second-order probability $\rho_{i,2} = 1$ and $\rho_{i,1} = \rho_{i,3} = 0$. Reducing $\rho_{i,2}$ and increasing $\rho_{i,1}$ and $\rho_{i,2}$ thus introduces ambiguity, and maximum ambiguity is reached at $\rho_{i,1} = \rho_{i,2} = \rho_{i,3} = 1/3$. The two extreme cases have identical overall variances if $\sigma_{i,1}^2 + \sigma_{i,3}^2 = 8\sigma_{i,2}^2$, where $\sigma_{i,k}$ is the standard deviation associated with $F_{i,k}$. In addition, ambiguity can also be increased

or reduced without affecting the overall expected processing time, even if $\mathcal{F}_i$ includes asymmetric distributions (e.g., exponential) with different means.

To consistently compare the efficiency of different ports, we scale $\tilde{e}_i$ and $\tilde{e}_j$ such that $E[\tilde{e}_i] = E[\tilde{e}_j]$ for any given $i \neq j$. In other words, the expected port processing times are all identical, but have different degrees of ambiguity with some ports having better-defined processing time distributions than others do.

3.2. Buyer's and Supplier's Objectives

The buyer may place an order with the supplier under two standard Incoterms: FOB or CIF. Under FOB, the supplier is responsible for delivering the goods to an origin port, while under CIF, the supplier must deliver the goods to a destination port (importing port) specified by the buyer. Under FOB, the supplier absorbs transportation costs only up to the exporting port i, whereas under CIF the supplier absorbs all of the transportation costs to the destination port.

To distinguish between these two arrangements, let α_i denote the fraction of the freight cost via port i. The supplier then incurs a freight cost of $\alpha_i(\phi_i + s_i Q)$ under FOB but incurs $\phi_i + s_i Q$ under CIF, where ϕ_i and s_i are the fixed and variable (unit) freight costs, respectively. The supplier's unit production cost is c, and it charges the buyer a unit selling price of w.

Under FOB, the buyer's objective is to determine an order quantity Q that maximizes its expected profit when port i is selected.

$$
\begin{aligned}
\pi_B(Q) =\, & (p_f E_D[\min\{Q, D\}] + p_d E_D[(Q - D)^+]) \\
& \cdot \mathrm{Prob}\{l_i + \tilde{e}_i \leq T\} + p_d Q \mathrm{Prob}\{l_i + \tilde{e}_i > T\} \\
& - hQ E_{\tilde{e}_i}[(t - l_i - \tilde{e}_i)^+] - wQ \\
& - (1 - \alpha_i)(\phi_i + s_i Q),
\end{aligned} \tag{1}
$$

where the first two terms are the expected revenue if the shipment arrives in time for the regular selling season or arrives late, respectively, and the last three terms capture the expected holding (if the shipment arrives early), purchasing, and transportation costs respectively. The inventory carrying-cost along the voyage is subsumed in the transportation cost s_i. If CIF is used, the last term is dropped from the above expression because the supplier absorbs all of the transportation costs.

The supplier's corresponding objective function is to determine the unit selling price w that maximizes its expected profit.

$$
\pi_S(w) = (w - c)Q - \alpha_i(\phi_i + s_i Q), \tag{2}
$$

where the first term captures the supplier's profit and the second term captures the transportation cost under FOB. If CIF is used, then the second term is

replaced by $\phi_i + s_i Q$, that is, the supplier bears all of the transportation costs.

In practice, the supplier can simply implement a cost-plus pricing approach to determine its unit selling price w. In such a case, the supplier can set $w = c + \alpha_i s_i + \gamma$, where γ is the supplier's desired unit margin. We will consider this alternative pricing scheme later.

Several factors are not captured in the above model but are worth discussing here. First, the supplier may be held liable for late shipments and the buyer may assess a penalty against the supplier if the shipment arrives after time T. However, if the shipment delay is caused by port processing time uncertainty, the supplier may claim force majeure, such as an interruption of the operations of a terminal accepting a tanker. A supplier that experiences an unusually lengthy customs clearance, such as due to a labor strike, can also claim force majeure, but for more usual situations the liabilities are industry-specific. Second, the supplier may also be concerned about cash flows involving accounts receivable and/or exchange rate risks that may influence the attractiveness of FOB vs. CIF as well as the supplier's selling price w. Our model does not capture such issues as our primary focus is on port processing time. Lastly, port delays may also generate additional port storage-related costs for the supplier, but we ignore such issues here as well.

4. Basic Structure

4.1. Buyer's Ordering Decision

In the following, we examine the buyer's objective function for any given unit purchasing price w (quoted by the supplier). Simplifying Equation (1), we have

$$
\begin{aligned}
\pi_B(Q) =\, & E_{\rho_{i,k}}\left[(p_f - p_d)\left(\int_{x \leq Q} x\, dG(x) + Q\overline{G}(Q) \right) \right. \\
& \cdot F_{i,k}(T - l_i) - hQ\left((t - l_i)F_{i,k}(t - l_i) \right. \\
& \left. \left. - \int_{\tilde{e}_i \leq t - l_i} \tilde{e}_i\, dF_{i,k}(\tilde{e}_i) \right) \right] - (w + (1 - \alpha_i) \\
& s_i - p_d)Q - (1 - \alpha_i)\phi_i.
\end{aligned} \tag{3}
$$

The second term in the square brackets drops out if $t \leq l_i$, in which case the shipment cannot arrive before the start of the selling season t. The following proposition proves that the buyer's order quantity decision is well-behaved.

PROPOSITION 1. *The buyer's expected profit function is concave in the order quantity Q. Furthermore, the*

optimal (interior) order quantity Q^* is given by the following:

$$\overline{G}(Q^*) = \frac{hE_{\rho_{i,k}}\left[(t-l_i)F_{i,k}(t-l_i) - \int_{\tilde{e}_i \leq t-l_i} \tilde{e}_i dF_{i,k}(\tilde{e}_i)\right] + w + (1-\alpha_i)s_i - p_d}{(p_f - p_d)E_{\rho_{i,k}}F_{i,k}(T-l_i)}. \quad (4)$$

The following sensitivity results ensue:

COROLLARY 1. *The buyer's optimal order quantity is (a) increasing in unit selling price p_f, discount price p_d, the supplier's share of transportation cost α_i, and the length of the selling season T; (b) decreasing in unit purchasing cost w, unit holding cost h, unit transportation cost s, and the starting time of the selling season t.*

The observation that the buyer orders larger quantities as T increases and smaller quantities as t increases warrants further discussion. Firstly, because we have fixed the starting time, that is, the ordering time, at time 0, an increase in T implies that the firm orders well in advance of the selling season and is hence more likely to receive the shipment in time. This reduced risk allows the buyer to order more. However, as the buyer places its order early, its demand estimate becomes less accurate, which may dampen its preference for early orders. Wang and Tomlin (2009) treat this problem in more detail.

In contrast, when t increases, the selling window $T - t$ narrows and, although this also implies that the buyer orders early with respect to the start of the season, the reduced selling time span exerts dominate effect on the buyer's expected profit, impelling the buyer to reduce its order to hedge against the higher risk of a limited selling time.

Analogously to Corollary 1, we can establish similar sensitivity results for the buyer's optimal expected profit through the envelope theorem. Hence the buyer's expected profit is intimately related to its order quantity, and we can therefore study how system parameters (i.e., processing time ambiguity) influence the buyer's expected profit by observing its order quantity decision.

4.2. Supplier's Pricing Decision
When the supplier quotes unit selling price w, it takes into consideration the buyer's ordering decision as described in section 4.1. As such, we use the notation $Q^*(w)$ to stress that the buyer's optimal order quantity depends on the supplier's price quote w. By Equation (2), we have

$$\pi_S(w) = (w - c - \alpha_i s_i)Q^*(w) - \alpha_i \phi_i. \quad (5)$$

The following assumption enables us to characterize the supplier's optimal pricing decision.

ASSUMPTION 1. *The demand distribution $G(\cdot)$ satisfies an increasing failure rate (IFR), that is, $g(x)/\overline{G}(x)$ weakly increases in x for all $G(x) < 1$.*

A wide range of distribution functions satisfy IFR, and Lariviere (2006) provides a detailed discussion of the IFR and increasing generalized failure rate (IGFR).[7] The following proposition proves that under mild assumptions the supplier's pricing decision problem is well behaved:

PROPOSITION 2. *Suppose Assumption 1 holds. If $p_d - c \leq s_i$, then the supplier's objective function is concave in the unit selling price w quoted to the buyer. Furthermore, the optimal unit selling price w^* is given by*

$$w^* = c + \alpha_i s_i + (p_f - p_d)g(Q^*)Q^* E_{\rho_{i,k}}\left[F_{i,k}(T - l_i)\right], \quad (6)$$

where Q^* is given by Proposition 1.

By Proposition 2, the supplier quotes a selling price that covers its unit production cost and its share of variable transportation costs, plus additional margins that depend on the buyer's profitability as well as the selling time T.

In practice, many suppliers still use the cost-plus approach in quoting unit selling prices. In other words, the suppliers may simply add a fixed percentage of margins to their cost (unit production and transportation) to arrive at their final price quotes. Although this practice is theoretically suboptimal, its ease of implementation leads to its widespread usage. The cost-plus approach can be seen as a special case of the optimal pricing decision described in Equation (6), where $w^*_{cp} = c + \alpha_i s_i + \gamma$ and the profit margin γ can be regarded as the supplier's expectations of $E_{\rho_{i,k}}[(p_f - p_d) Q^* g(Q^*)F_{i,k}(T - l_i)]$. We will highlight the impact of the cost-plus pricing approach on the buyer's and supplier's preferences over (or tolerance of) processing time ambiguity.

4.3. Equilibrium Behaviors
Given the buyer's and supplier's optimal ordering and pricing decisions, we can further characterize their equilibrium behaviors. Substituting Equation (6) into Equation (4), we have

$$\overline{G}(Q^*) = Q^* g(Q^*)$$

$$\frac{hE_{\rho_{i,k}}\left[(t-l_i)F_{i,k}(t-l_i) - \int_{\tilde{e}_i \leq t-l_i} \tilde{e}_i dF_{i,k}(\tilde{e}_i)\right]}{+ \frac{+ c + s_i - p_d}{(p_f - p_d)E_{\rho_{i,k}}F_{i,k}(T - l_i)}}. \quad (7)$$

It follows directly from Equation (7) that the buyer's optimal order quantity does not depend on its share of transportation costs α_i. This suggests that regardless of whether its transaction is based on FOB or CIF, all else being equal the buyer's order quantity will remain the same. This is consistent with casual observations in practice that firms rarely adjust their order quantity in response to changes in Incoterms.

In contrast, combining Equation (7) with Equation (6), we find that the supplier's pricing decision w does depend on its share of transportation costs. Nevertheless, the supplier's optimal selling price w^* fully absorbs transportation costs, such that regardless of whether FOB or CIF is used, the supplier's expected profit is also unaffected by Incoterms.

The above observations are based on the assumption that the supplier has full pricing power. In case the supplier practices the cost-plus pricing scheme described at the end of section 4.2, such that the unit price equals a fixed proportion of the production cost plus transportation costs, that is, $w = c + \alpha_i s_i + \gamma$, then both the supplier's unit price and the buyer's order quantity are influenced by Incoterms. As a result, their expected profits are also affected by Incoterms.

5. Impact of Lead Time Ambiguity

Increased ambiguity in port processing time affects the port's attractiveness to the buying firm by influencing the firm's subjective assessment of the likelihood that the shipment will arrive in time for the selling season. The supplier's preference is also indirectly affected by ambiguity levels through the buying firm's ordering behavior.

If increased ambiguity is associated with a longer expected mean processing time, the buying firm understandably finds the port less attractive and *vice versa*. In contrast, the buying firm's preferences are less obvious when the expected mean processing time remains constant as ambiguity levels vary. It is not immediately clear a priori whether the buying firm always prefers a port with less ambiguity, given the same mean processing time. In the following, we focus on the latter case where the expected mean processing time is kept constant while the ambiguity levels are varied.

To place our study of ambiguity into practical context, let us first consider two different scenarios where ambiguity in port processing time may arise.

1. The port processing time is influenced by discrete, isolated events, but the firm is unsure which events will occur. For example, the shipment may or may not by delayed at customs due to various operational factors. If there is no delay, the shipment's processing time $\tilde{e}_i$ is characterized by $F_{i,1}(\cdot)$; if there is a delay, the processing time $\tilde{e}_i$ is characterized by $F_{i,2}(\cdot)$. In this example, it is expected that $E_{F_{i,1}(\cdot)}[\tilde{e}_i] < E_{F_{i,2}(\cdot)}[\tilde{e}_i]$, and ambiguity mainly influences the firm's assessment of the expected port processing time.

2. The port processing time is influenced by frequent, random noise. For example, the heterogeneity of the mix of shipments at the port varies over time. These varying cargo mixes may influence the variability of the port processing time but not necessarily the expected processing time. In such cases ambiguity mainly influences the firm's assessment of the variability in the port processing time.

The above examples lead to two natural classifications of processing time ambiguity based on whether the distributions (in the ambiguity set $\mathcal{F}_i$) satisfy first-order stochastic dominance (FOSD) or second-order stochastic dominance (SOSD). Figures 2 and 3 illustrate examples of these two types of processing time ambiguity.

Figure 2 illustrates an ambiguity set $\mathcal{F}_i$ within which the three distributions satisfy the FOSD, that is, $E_{F_{i,1}}[\tilde{e}_i] < E_{F_{i,2}}[\tilde{e}_i] < E_{F_{i,3}}[\tilde{e}_i]$. The set of distributions in $\mathcal{F}_i$ need not have identical variances. The anticipated mean processing time for port i depends on $\rho_{i,k}$, the firm's subjective assessment of the likelihood of $\tilde{e}_i$ being drawn from $F_{i,k}$.

In contrast, Figure 3 illustrates an ambiguity set within which the three distributions satisfy the SOSD. The set of distributions in $\mathcal{F}_i$ have identical mean but different variances, and, as a result, the anticipated mean processing time does not depend on $\rho_{i,k}$ but the anticipated variance of $\tilde{e}_i$ does. A mean-preserving spread of the set of distributions in $\mathcal{F}_i$ reflects increased processing time uncertainty but not ambiguity.

Below, we first explore the ambiguity set with FOSD and then that with SOSD, assuming that the buyer is ambiguity neutral. We will consider

Figure 2   Illustration of the Ambiguity Set with Distributions Satisfying FOSD [Color figure can be viewed at wileyonlinelibrary.com]
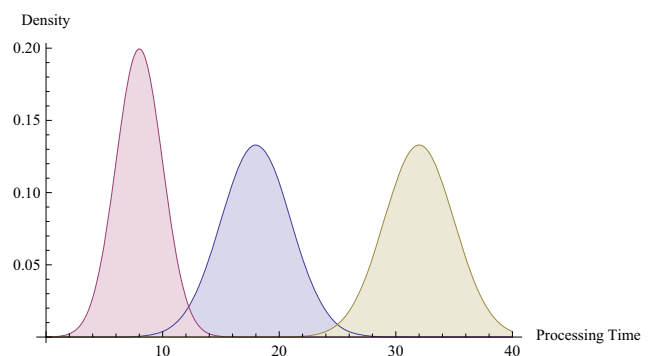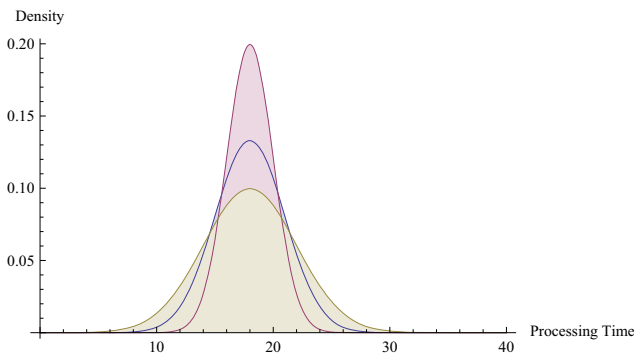
Figure 3 Illustration of the Ambiguity Set with Distributions Satisfying SOSD [Color figure can be viewed at wileyonlinelibrary.com]

ambiguity aversion in section 5.3. For expositional ease, in the following we assume that $M_i = |\mathcal{F}_i|$ is an odd number, and define $\overline{M}_i = (1 + M_i)/2$, that is, $\overline{M}_i$ indexes the middle distribution in $\mathcal{F}_i$.

5.1. Lead Time Ambiguity Manifested through FOSD

By the definition of FOSD, the distributions in the ambiguity set $\mathcal{F}_i$ satisfy $F_{i,k}(x) > F_{i,l}(x)$ for any $1 \leq k < l \leq M_i$ and any given x. To facilitate the further characterization of the buyer's and supplier's optimal behavior, we also need to consider the firm's subjective assessment of $\tilde{e}_i$ being drawn from $F_{i,k}$, that is, the structure of $\rho_{i,1}, \rho_{i,2}, \ldots, \rho_{i,M_i}$. A natural structure that can represent values of $\rho_{i,k}$ is the discrete symmetric unimodal distribution $\rho_i(k, a)$, where $\rho_{i,k} = \rho_i(k, a)$ and $a \in \{\xi | 1/M_i \leq \xi \leq 1\}$ measures the height (i.e., dispersion) of the $\rho_i(\cdot, a)$ function. In particular, $a = 1$ reduces the processing time to a *predictable* one because it implies that $\rho_{i,\overline{M}_i} = 1$ (and $\rho_{i,k \neq \overline{M}_i} = 0$) due to the unimodal assumption. In contrast, $1/M_i \leq a < 1$ corresponds to an *ambiguous* processing time distribution. In the special case of $a = 1/M_i$, the processing time $\tilde{e}_i$ is equally likely to be drawn from any distribution from the ambiguity set $\mathcal{F}_i$.

The $\rho_i(\cdot, a)$ function behaves somewhat, but not exactly, like a tent map function, where a larger value of a corresponds to more concentrated and hence more predictable, subjective assessments, whereas a smaller value of a corresponds to more dispersed and ambiguous, subjective assessments. It follows from the above definition that for any given a and $1 \leq k < \overline{M}_i$, we have $\rho_i(k, a) = \rho_i(M_i - k + 1, a)$. Similarly, we have $\rho_i(k, a) < \rho_i(l, a)$ for any $k < l < \overline{M}_i$, and $\rho_i(k, a) > \rho_i(l, a)$ for any $\overline{M}_i > k > l$.

Leveraging the above observations, we define a useful metric to characterize the impact of ambiguity. For a given port i, an anticipated probability of the lead time being less than x under ambiguity level a is defined as

$$L_i(x, a) = \sum_{k=1}^{M_i} \rho_i(k, a) F_{i,k}(x)$$
$$= \sum_{k=1}^{\overline{M}_i - 1} \rho_i(k, a)\big(F_{i,k}(x) + F_{i,M_i-k+1}(x)\big) \qquad (8)$$
$$+ \rho_i(\overline{M}_i, a) F_{i,\overline{M}_i}(x).$$

By the definition of $\rho_i(\cdot, a)$, we have $\partial \rho_i(\overline{M}_i, a)/\partial a > 0$ and $\partial \rho_i(k, a)/\partial a < 0$ for some $k < \overline{M}_i$. Notice that

$$\frac{\partial L_i(x, a)}{\partial a} = \sum_{k=1}^{\overline{M}_i - 1} \frac{\partial \rho_i(k, a)}{\partial a} \big(F_{i,k}(x) + F_{i,M_i-k+1}(x)\big)$$
$$+ \frac{\partial \rho_i(\overline{M}_i, a)}{\partial a} F_{i,\overline{M}_i}(x). \qquad (9)$$

Although the second term in Equation (9) is always positive, the first term may be either positive or negative. Hence it is not immediately obvious whether $L_i(x, a)$ is monotone in a in general. With some regulatory conditions, however, the following lemma characterizes the behavior of the $L_i(x, a)$ function.

LEMMA 1. *For a given $x > 0$, if*

$$F_{i,k}(x) + F_{i,M_i-k+1}(x) \geq F_{i,l}(x) + F_{i,M_i-l+1}(x) \geq F_{i,\overline{M}_i}(x) \qquad (10)$$

for all $1 \leq k < l < \overline{M}_i$, then $\partial L_i(x, a)/\partial a \geq 0$. Otherwise if the inequality sign in Equation (10) is reversed then $\partial L_i(x, a)/\partial a < 0$.

It is worth pointing out that the results in Lemma 1 depend on the value of x, as different values of x in general influence the inequality condition (10). Leveraging Lemma 1, we can now investigate how lead time ambiguity affects the buyer's and supplier's preference toward a particular port. Substituting $x = T - l_i$ into the $L_i(x, a)$ function, we have $L_i(x, a) = L_i(T - l_i, a) = E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]$, where a larger value of a corresponds to a less ambiguous subjective assessment of the lead time distribution. To illustrate how $E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]$ changes with the ambiguity level a (in relation to condition (10) in Lemma 1), we consider two special cases where $F_{i,k}$ follows the uniform distribution and the exponential distribution.

The uniform distribution is appropriate when the firm is minimally informed about the processing time distribution except plausible lower and upper bounds. The firm then relies on second-order probability $\rho_{i,k}$ to assess the likelihood that a particular set of (lower and upper) bounds is correct.

In contrast, the exponential distribution is more appropriate if the firm knows a set of plausible values of the *mean* processing time. The firm then relies on $\rho_{i,k}$ to assess the likelihood that a particular value for the mean processing time is correct. The exponential distribution guarantees that the processing time $\tilde{e}_i$ is positive, and because it has long tails, the exponential distribution is often an accurate approximation of $\tilde{e}_i$ when only the mean processing time is specified.

5.1.1. Ambiguity Set with Uniform Distribution.

Suppose $|\mathcal{F}_i| = 3$ and $F_{i,k} \sim U(\mu_{i,k}, \sigma_{i,k})$, where $\mu_{i,1} < \mu_{i,2} < \mu_{i,3}$ and $\sigma_{i,1} = \sigma_{i,2} = \sigma_{i,3} = \sigma_i$, such that $E_{F_{i,k}}[\tilde{e}_i]$ increases with k but the variance of $\tilde{e}_i$ does not depend on k. This ensures that specifications of $F_{i,k}$ indeed satisfy FOSD. For clarity, assume that the distributions have positive support and do not overlap, that is, $\mu_{i,k} - \sqrt{3}\sigma_i \geq \mu_{i,k-1} + \sqrt{3}\sigma_i \geq 0$. The second-order probabilities are $\rho_i(1, a) = \rho_i(3, a) = \frac{1}{2}(1 - \rho_i(2, a)) = \frac{1}{2}(1 - a)$, where $\frac{1}{3} \leq a \leq 1$. We have

$$E_{\rho_{i,k}}\left[F_{i,k}(T - l_i)|a\right] = L_i(T - l_i, a)$$
$$= \rho_i(1,a)\left(F_{i,1}(T - l_i) + F_{i,3}(T - l_i)\right) + \rho_i(2,a)F_{i,2}(T - l_i)$$
$$= \frac{1}{2}(1 - a)\left\{ \min\left(\frac{(T - l_i - \mu_{i,1} + \sqrt{3}\sigma_i)^+}{2\sqrt{3}\sigma_i}, 1\right) \right.$$
$$\left. + \min\left(\frac{(T - l_i - \mu_{i,3} + \sqrt{3}\sigma_i)^+}{2\sqrt{3}\sigma_i}, 1\right) \right\}$$
$$+ a\min\left(\frac{(T - l_i - \mu_{i,2} + \sqrt{3}\sigma_i)^+}{2\sqrt{3}\sigma_i}, 1\right). \quad (11)$$

Notice that the above expression satisfies condition (10) in both ways for any given $T - l_i$, and hence the sign of $\partial E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]/\partial a$ cannot be solely determined by specifications of $F_{i,k}(\cdot)$. As mentioned above, the sign depends on the value of $T - l_i$. Suppose $T - l_i$ is within the maximum range of the lead time distributions, that is, $\mu_{i,1} - \sqrt{3}\sigma_i \leq T - l_i \leq \mu_{i,3} + \sqrt{3}\sigma_i$.

If $\mu_{i,2} - \sqrt{3}\sigma_i \leq T - l_i \leq \mu_{i,2}$, then by Equation (11) we have

$$\frac{\partial E_{\rho_{i,k}}\left[F_{i,k}(T - l_i)|a\right]}{\partial a} = -\frac{1}{2} + \frac{T - l_i - \mu_{i,2} + \sqrt{3}\sigma_i}{2\sqrt{3}\sigma_i} \leq 0. \quad (12)$$

It can be verified that $\partial E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]/\partial a \leq 0$ for any $T - l_i \leq \mu_{i,2}$ and the sign is reversed for any $T - l_i > \mu_{i,2}$.

The above example illustrates that with uniform distributions, increased ambiguity (i.e., more diffuse second-order probability) leads to a shorter *anticipated* lead time if $T - l_i \leq \mu_{i,2}$ and *vice versa*. In the following, we use $\bar{\mu}_i$ (which equals $\mu_{i,2}$ in the above example)

to denote the *midpoint processing time* of port i. The following proposition ensues.

PROPOSITION 3. *Suppose $F_{i,k} \in \mathcal{F}_i$ follows a uniform distribution, and $t \leq l_i$ or $h \approx 0$. If $T - l_i \leq \bar{\mu}_i$ then as the processing time becomes more ambiguous,*

(a) *The buyer's optimal order quantity Q^* increases;*
(b) *The supplier quotes a higher unit selling price w^*;*
(c) *Both the supplier's and buyer's optimal expected profits increase;*

The reverse is true if $T - l_i > \bar{\mu}_i$.

The buyer's and supplier's preference towards a particular port therefore depends critically on how important it is for the shipment to meet the selling time window. For time-sensitive shipments, where the midpoint processing time is greater than the transit buffer time, that is, $\bar{\mu}_i > T - l_i$, a port with an ambiguous processing time is more appealing because there is a greater chance that the processing time will turn out to be short so that the shipment arrives in time for the selling season. Although there is also an equally greater chance that the shipment will arrive late, the upside gain from its arriving in time dominates the downside loss from its arriving late.[8] In contrast, with little ambiguity, the shipment is almost guaranteed to be late for the selling season. Under practical settings where the shipment is planned ahead with sufficient transit buffer time, that is, $\bar{\mu}_i \leq T - l_i$, the buying firm is likely to prefer a port with less ambiguity in processing time. Hence, buyers and suppliers that plan ahead will more likely choose ports with less ambiguous processing times.

Regardless of whether the shipment is time-sensitive, that is, whether $\bar{\mu}_i > T - l_i$, the buyer and the supplier are equally affected by ambiguity: they either both benefit from increased ambiguity or are both hurt by it. Combining this observation with the earlier result that both the buyer and the supplier are indifferent to Incoterms (see section 4.3), we can conclude that the buyer's and supplier's preferences toward ambiguity are not affected byIncoterms—as long as the supplier optimizes the unit wholesale price w.

Such consistent preferences have significant managerial implications. Because Incoterms have no perceivable impact on buyer and supplier preferences, a port's attractiveness is significantly influenced by its overall transit buffer time and processing time ambiguity.

For example, although the port of Shenzhen is closer to the hinterland of the PRD than that of Hong Kong, if the overall transit buffer time $T - l_i$ is the same their attractiveness to buyers or suppliers is largely influenced by whether the shipment is

time-sensitive and their relative operational efficiency in processing the shipment. Suppose hypothetically that the Shenzhen port exhibits more ambiguity than the Hong Kong port. All else being equal both the buyer and the supplier are likely to select the Shenzhen port for time-sensitive shipments but pick the Hong Kong port for most regular shipments. The Shenzhen port could therefore attract more regular shipments by reducing the processing time ambiguity and enhancing operational efficiency. In contrast, a more reasonable approach for the Hong Kong port to attract more shipments is to further reduce its midpoint processing time $\bar{\mu}_i$ while maintaining or reducing the associated ambiguity levels.

The above analysis thus far ignores inventory holding costs if a shipment arrives early, which will dampen the directional effect discussed above. Our subsequent numerical study (section 6) suggests that such a dampening effect does not qualitatively impact the above observations.

5.1.2. Ambiguity Set with Exponential Distribution.
The exponential distribution is more realistic in practice, as processing times often adhere to certain published standards but can be significantly longer (i.e., with long tails) when there are delays. Suppose $|\mathcal{F}_i| = 3$ and $F_{i,k} \sim \exp(\mu_{i,k})$, where $\mu_{i,1} < \mu_{i,2} < \mu_{i,3}$ such that $F_{i,k}$'s satisfy FOSD. The second-order probabilities are $\rho(1, a) = \rho(3, a) = \frac{1}{2}(1 - \rho(2, a)) = \frac{1}{2}(1 - a)$, where $\frac{1}{3} \le a \le 1$. We have

$$
\begin{aligned}
E_{\rho_{i,k}}\big[F_{i,k}(T - l_i)|a\big] &= L_i(T - l_i, a) = \rho_i(1, a)\big(F_{i,1}(T - l_i) \\
&\quad + F_{i,3}(T - l_i)\big) + \rho_i(2, a)F_{i,2}(T - l_i) \\
&= \frac{1}{2}(1 - a)\left\{1 - \exp\left(-\frac{T - l_i}{\mu_{i,1}}\right)\right. \\
&\quad \left. + 1 - \exp\left(-\frac{T - l_i}{\mu_{i,3}}\right)\right\} \\
&\quad + a\left\{1 - \exp\left(-\frac{T - l_i}{\mu_{i,2}}\right)\right\}. \quad (13)
\end{aligned}
$$

Directly verifying the above expression with condition (10) in Lemma 1 is analytically challenging. The following lemma, however, partially characterizes the behavior of $E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]$.

LEMMA 2. *Suppose values of $\mu_{i,j}$ are symmetric such that they can be represented as $\mu_{i,1} = \mu - x$, $\mu_{i,2} = \mu$, and $\mu_{i,3} = \mu + x$. Define $\theta(k, x) = (e^{-\frac{k}{\mu+x}} - e^{-\frac{k}{\mu}})/(e^{-\frac{k}{\mu+x}} - e^{-\frac{k}{\mu-x}})$. If $\theta(T - l_i, x) \ge 1/2$, then $\partial E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]/\partial a \ge 0$, and the reverse is true if $\theta(T - l_i, x) < 1/2$. Furthermore, for any given x, there exists a $\bar{k}$ such that $\theta(k, x) \ge 1/2$ for any $k \ge \bar{k}$, and $\theta(k, x) < 1/2$ for any $k < \bar{k}$.*

Similarly to the uniform distribution case, $E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]$ can be increasing or decreasing in a, which depends partially on the shipping buffer time $T - l_i$. Specifically, if the shipping buffer time exceeds a critical threshold then $E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]$ always increases in a. Leveraging Lemma 2, the following proposition describes the impact of ambiguity on port preference when the ambiguity set $\mathcal{F}_i$ consists of exponential distributions.

PROPOSITION 4. *Suppose $F_{i,k} \in \mathcal{F}_i$ is exponentially distributed, and that $t \le l_i$ or $h \approx 0$. If the shipping buffer time $T - l_i$ is sufficiently large (i.e., $T - l_i \ge \bar{k}$), then as the processing time becomes more ambiguous,*

- (a) *The buyer's optimal order quantity Q^* decreases;*
- (b) *The supplier quotes a lower unit selling price w^*;*
- (c) *Both the supplier's and buyer's optimal expected profits decrease.*

The reverse is true if $T - l_i < \bar{k}$.

The supplier's and buyer's preferences therefore exhibit similar traits regardless of whether the ambiguity set consists of uniform or exponential distributions. Ambiguity can influence the supplier's and the buyer's choice of port in different ways. If a shipment is not time-sensitive both the supplier and the buyer tend to prefer a port with less ambiguity. An important managerial insight is that a port can increase its competitiveness by reducing the ambiguity associated with its processing times for regular shipments. This can even enable it to overcome a locational disadvantage, such as longer distances to the hinterland or shorter transit buffer times ($T - l_i$), to gain business from both the supplier and the buyer by improving its processing predictability for regular shipments.

On the other hand, if a shipment is time-sensitive, their preferences reverse and they tend to prefer a port with *more* ambiguity. Part of the intuition is that the realized processing time can equally be short or long, but the benefit from a short processing time (thus allowing the shipment to arrive in time for the selling season) offsets the cost of a long processing time. An important caveat is that, as we will see in our subsequent numerical study (section 6), ambiguity can affect both regular and time-sensitive shipments with asymmetrical impacts on the buyer and the supplier's expected profit, and this impact depends on the distributional assumption in the ambiguity set.

The above observations are based on $|\mathcal{F}_i| = 3$, that is, the ambiguity set contains three plausible distributions. While it has a natural interpretation in practice, that is, firms often rely on estimations for three processing time scenarios (optimistic, most likely, and pessimistic), it is unclear whether the above

observations continues to hold when the ambiguity set increases. We address this question in section 6.

5.2. Lead Time Ambiguity Manifested through SOSD

By the definition of SOSD, distributions in ambiguity set $\mathcal{F}_i$ possess different variances but the same mean processing time. In practice, this implies that the firm knows the mean processing time but is unsure of the correct variance.

As noted in section 3, it is important to realize that increased ambiguity may not necessarily correspond to increased variance. In particular, increased ambiguity implies that distributions with both larger and smaller variances are more likely to be true and hence the overall variance may not necessarily increase.

With SOSD, we stipulate that the set of distributions in $\mathcal{F}_i$ have the same mean and the distributions cross only once, that is, they possess a single-crossing property. Such a set of distributions can be generated through a mean-preserving spread approach. For the second-order probabilities we adopt a similar structure and notation to those described in the FOSD case (see section 5.1).

As in the FOSD case, for a given port i we define the anticipated probability of the processing times being less than x under ambiguity level a as

$$
L_i(x,a) = \sum_{k=1}^{\overline{M}_i - 1} \rho_i(k,a)\big(F_{i,k}(x) + F_{i,M_i-k+1}(x)\big) + \rho_i(\overline{M}_i, a) F_{i,\overline{M}_i}(x).
\tag{14}
$$

By the definition of $\rho_i(\cdot, a)$, we have $\partial \rho_i(\overline{M}_i, a)/\partial a > 0$ and $\partial \rho_i(k, a)/\partial a < 0$ for some $k < \overline{M}_i$. Notice that

$$
\frac{\partial L_i(x,a)}{\partial a} = \sum_{k=1}^{\overline{M}_i - 1} \frac{\partial \rho_i(k,a)}{\partial a}\big(F_{i,k}(x) + F_{i,M_i-k+1}(x)\big) + \frac{\partial \rho_i(\overline{M}_i, a)}{\partial a} F_{i,\overline{M}_i}(x).
\tag{15}
$$

Similar to the FOSD case, although the second term in Equation (15) is always positive, the first term may be either positive or negative. Hence it is not immediately obvious whether the $L_i(x, a)$ function is monotone in a in general. It turns out that Lemma 1 continues to apply in the SOSD case, that is, the sign of Equation (15) depends on the relative magnitude of $F_{i,k}(x) + F_{i,M_i-k+1}(x) \sim F_{i,l}(x) + F_{i,M_i-l+1}(x) \sim F_{i,\overline{M}_i}(x)$, as well as the value of x involved.

Leveraging Lemma 1, we can investigate how processing time ambiguity affects the buyer's and supplier's preference toward a particular port. Substituting $x = T - l_i$ into the $L_i(x, a)$ function, we have $L_i(x, a) = L_i(T - l_i, a) = E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]$, where a

higher a corresponds to a more predictable processing time. To illustrate how $E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]$ changes with ambiguity level a (in relation to condition (10) in Lemma 1), we consider the special case of an ambiguity set with $|\mathcal{F}_i| = 3$ where $F_{i,k} \in \mathcal{F}_i$ follows a uniform distribution. Note that the exponential distribution does not apply in the SOSD case because it is asymmetric.

Suppose $F_{i,k} \sim U(\mu_{i,k}, \sigma_{i,k})$, where $\mu_{i,1} = \mu_{i,2} = \mu_{i,3} = \mu_i$ and $\sigma_{i,1} > \sigma_{i,2} > \sigma_{i,3}$ such that $F_{i,k}$'s satisfy SOSD. The second-order probabilities are $\rho(1, a) = \rho(3, a) = \frac{1}{2}(1 - \rho(2, a)) = \frac{1}{2}(1 - a)$, where $\frac{1}{3} \le a \le 1$. We have

$$
\begin{aligned}
E_{\rho_{i,k}}\big[F_{i,k}(T - l_i)|a\big] &= L_i(T - l_i, a) = \rho_i(1,a)\big(F_{i,1}(T - l_i) \\
&\quad + F_{i,3}(T - l_i)\big) + \rho_i(2,a) F_{i,2}(T - l_i) \\
&= \frac{1}{2}(1 - a)\left\{ \min\left(\frac{(T - l_i - \mu_i + \sqrt{3}\sigma_{i,1})^+}{2\sqrt{3}\sigma_{i,1}}, 1\right) \right. \\
&\quad \left. + \min\left(\frac{(T - l_i - \mu_i + \sqrt{3}\sigma_{i,3})^+}{2\sqrt{3}\sigma_{i,3}}, 1\right) \right\} \\
&\quad + a \min\left(\frac{(T - l_i - \mu_i + \sqrt{3}\sigma_{i,2})^+}{2\sqrt{3}\sigma_{i,2}}, 1\right).
\end{aligned}
\tag{16}
$$

The above expression satisfies condition (10) when $T - l_i \le \mu_i - \sqrt{3}\sigma_{i,1}$ (or when $T - l_i > \mu_i + \sqrt{3}\sigma_{i,1}$), and the sign is reversed otherwise. Hence, by Lemma 1, we have $\partial E_{\rho_{i,k}}[F_{i,k}(T - l_i)|a]/\partial a \le 0$ when $T - l_i$ is very small or moderately large; otherwise when $T - l_i$ is moderately small or very large the sign is reversed. The following lemma formalizes the above observation.

LEMMA 3. *Suppose $F_{i,k} \in \mathcal{F}_i$ follows the uniform distribution satisfying SOSD. If $x \le \mu_i - \sqrt{3}\sigma_{i,1}\sigma_{i,2}/(2\sigma_{i,1} - \sigma_{i,2})$ or $x > \mu_i + \sqrt{3}\sigma_{i,1}\sigma_{i,2}/(2\sigma_{i,1} - \sigma_{i,2})$ then $\partial L_i(x, a)/\partial a \le 0$; otherwise if $\mu_i - \sqrt{3}\sigma_{i,1}\sigma_{i,2}/(2\sigma_{i,1} - \sigma_{i,2}) < x \le \mu_i + \sqrt{3}\sigma_{i,1}\sigma_{i,2}/(2\sigma_{i,1} - \sigma_{i,2})$ the sign is reversed.*

Lemma 3 suggests that the effect of SOSD differs from that of FOSD in the sense that the directional change of $L_i(x, a)$ cannot be unequivocally ranked by ambiguity level, as the sign of $\partial L_i(x, a)/\partial a$ changes from region to region, depending on how time sensitive a shipment is relative to the transit buffer time $T - l_i$.

While it is straightforward to enumerate the directional effects of ambiguity on the firm's preference (analogous to Proposition 3) by examining different levels of shipment time sensitivity, such levels continue to increase as $|\mathcal{F}_i|$ increases. This leads to a system in which the firm's preference towards ambiguity oscillates as the ambiguity level increases. In practice, therefore, it is neither useful nor feasible to develop

managerial insights on how SOSD ambiguity influences firms' preferences. As such, we conduct a numerical study in section 6 to investigate the effect of ambiguity under more general settings.

5.3. Aversion to Ambiguity

In the preceding sections we have assumed that decision-makers are ambiguity-neutral. In practice, decision-makers can be ambiguity averse (Klibanoff et al. 2005, Segal 1987). Following the smooth decision-making framework developed by Klibanoff et al. (2005), we can tailor (3) to incorporate the buyer's ambiguity attitude. For notational ease, define the buyer's utility function as

$$
U_{F_{i,k}}(Q) = (p_f - p_d)\left(\int_{x \leq Q} x dG(x) + Q\overline{G}(Q)\right)F_{i,k}(T - l_i)
$$
$$
- hQ\left((t - l_i)F_{i,k}(t - l_i) - \int_{\tilde{e}_i \leq t - l_i} \tilde{e}_i dF_{i,k}(\tilde{e}_i)\right).
\tag{17}
$$

The buyer's objective (under ambiguity aversion) can be expressed as

$$
\pi_B(Q) = E_{\rho_{i,k}}\left[\Theta(U_{F_{i,k}}(Q))\right]
$$
$$
- (w + (1 - \alpha_i)s_i - p_d)Q - (1 - \alpha_i)\phi_i,
\tag{18}
$$

where $\Theta(x)$ is an increasing concave function. Analogously to Proposition 1, the following proposition characterizes the buyer's objective under ambiguity aversion.

PROPOSITION 5. *The buyer's expected profit function (under ambiguity aversion) is concave in the order quantity Q. Furthermore, the optimal (interior) order quantity Q^* satisfies the following:*

$$
E_{\rho_{i,k}}\left[\Theta'(U_{F_{i,k}}(Q))U'_{F_{i,k}}(Q)\right] = w + (1 - \alpha_i)s_i - p_d. \tag{19}
$$

If $\Theta(\cdot)$ is linear then $\Theta'(\cdot)$ is constant, that is, the optimal order quantity is not influenced by the buyer's attitude toward ambiguity. In particular, setting $\Theta(x) = x$ recovers our base model in which the firm is ambiguity-neutral.

The following corollary shows that increased ambiguity aversion reduces the buyer's optimal order quantity. For clarity (and concreteness), in the following, we assume that $\Theta(x) = -e^{-\eta x}$ with $\eta \geq 0$, which is concave-increasing and has a constant coefficient of ambiguity $\eta = -\Theta''(x)/\Theta'(x)$. A higher η corresponds to a higher degree of ambiguity aversion.

COROLLARY 2. *If $\eta \geq 1$ then $\partial Q^*/\partial \eta \leq 0$, that is, the buyer's optimal order quantity decreases as ambiguity aversion increases.*

If the buyer is only slightly or moderately averse to ambiguity, e.g., $0 \leq \eta < 1$, then its optimal order quantity can be higher than if it is ambiguity-neutral. As the buyer becomes increasingly ambiguity-averse, however, the optimal order quantity eventually declines.

The supplier's objective function is influenced by ambiguity aversion through the buyer's ordering decisions. For notational convenience, let

$$
z_{i,k}(x) = \frac{d\Theta(U_{F_{i,k}}(x))}{dx} = \eta e^{-\eta U_{F_{i,k}}(x)}U'_{F_{i,k}}(x). \tag{20}
$$

The following proposition partially characterizes the supplier's optimal pricing decision under ambiguity aversion.

PROPOSITION 6. *If $z''_{i,k}(x) \geq 0$ then the supplier's objective function is concave in the unit selling price w quoted to the buyer. Furthermore, the optimal unit selling price w^* satisfies*

$$
w^* = c + \alpha_i s_i - Q^* E_{\rho_{i,k}}\left[z'_{i,k}(Q^*)\right], \tag{21}
$$

where Q^* is given by Proposition 5.

Note that $z'_{i,k}(Q^*) = \Theta''(U_{i,k}(Q^*))(U'_{i,k}(Q^*))^2 + \Theta'(U_{i,k}(Q^*))U''_{i,k}(Q^*) \leq 0$. The condition in Proposition 6 is not straightforward to verify, but a sufficient condition is

$$
\eta^2\left(U'_{i,k}(x)\right)^3 - 3\eta U'_{i,k}(x)U''_{i,k}(x) + U'''_{i,k}(x) \geq 0, \tag{22}
$$

in which the first two terms are both positive. Recognizing that $U'''_{i,k}(x) = -(p_f - p_d)g'(x)F_{i,k}(T - l_i)$, a (strong) sufficient but not necessary condition is $g'(x) \leq 0$. The supplier's optimal pricing decision can, however, still be well-behaved even if the condition is not satisfied.

While it is straightforward to establish that, under certain conditions, the optimal order quantity Q^* decreases in w, it is unclear whether the optimal w^* increases or decreases as the buyer's ambiguity aversion increases. As a result, it is not immediately clear how ambiguity aversion affects the buyer's and supplier's expected profits. We explore this question through a numerical study in section 6.

6. Numerical Study

The purpose of the numerical study is threefold. First, we relax the assumption that the ambiguity set contains three distributions ($|\mathcal{F}_i| = 3$) made in the analytical work. We also incorporate positive inventory holding costs with the possibility that the shipment will arrive on time. These relaxations allow us to test

the robustness of our analysis and examine the effect of a larger ambiguity set on firms' preferences. Second, we examine the magnitude of the ambiguity effect to gain a more realistic sense of the importance of ambiguity to profitability. Third, we examine the effect of ambiguity relative to other important parameters, especially the selling time window $T - t$ and the transit buffer time $T - l_i$, which will allow us to further illustrate the effect of ambiguity in a more practical setting.

In our numerical study, we operationalize the second-order probabilities (values of $\rho_{i,k}$) through the Beta distribution with parameters α and β. We set $\alpha = \beta$ such that the weights follow a symmetric, unimodal distribution. The reason we choose the Beta distribution is that this distribution is bounded between 0 and 1, which provides a natural domain for assigning second-order probabilities to $F_{i,k}$. Throughout the numerical study, we set $|\mathcal{F}_i| = 5$, that is, the ambiguity set contains five distributions. We implemented $\rho_{i,k}$ through the Beta distribution by varying α and β in lockstep from 1 to 3, 7, 15, and 31 (see Figure 4). Hence we have five ambiguity levels: $\alpha = \beta = 1$ is associated with the most ambiguous processing time because the values of $\rho_{i,k}$ are uniform, whereas $\alpha = \beta = 31$ corresponds to the least ambiguous processing time because the values of $\rho_{i,k}$ are most concentrated (high kurtosis).

For each value of α (and β), we use the equidistant set of discrete points $x \in (0.1, 0.3, 0.5, 0.7, 0.9)$ within the domain of the Beta distribution to obtain raw weights for $\rho_{i,k} = \mathcal{B}(x_k)$ for $k \in (1, 2, \ldots, 5)$. The values of $\rho_{i,k}$ are then normalized to $\Sigma_k \rho_{i,k} = 1$.

For market demand, we use the normal distribution and fix the parameters at $\mu_d = 100$ and $\sigma_d = 30$. For the transit lead time parameters, we set $T = 20$ and vary t from 13 to 17 with a step size of one such that the selling window varies from seven to three

Figure 4   Implementing $\rho_{i,k}$ through Beta Distribution (α and β Vary in Lockstep from 1 to 3, 7, 15, and 31) [Color figure can be viewed at wileyonlinelibrary.com]

periods. We also vary the total transit lead time (in the absence of the port processing time) l_i from eight to 12 with a step size of one such that the transit buffer time (to allow port processing) varies from 12 to eight.

For the financial parameters, we fix the selling price $p_f = 10$ and the discounted price $p_d = 3.0$. We vary the unit production cost c from 1.0 to 3.0 with a step size of 0.5. We set the unit inventory holding cost $h = 5\% \times c$ and the unit freight cost $s_i = 7\% \times c$. Note that the average transportation cost as a percentage of sales is 7.87% (Ellis 2014).[9] We set the fixed freight cost $\phi_i = 0$, because its directional impact is straightforward and it does not influence the firm's attitude toward processing time ambiguity. The parameter α_i for the supplier's share of shipping costs is set at 0.2 and 0.8.

In the FOSD case, we distinguish between uniform and exponential distributions. For the uniform distribution, we allow the $E_{F_{ik}}[\tilde{e}_i]$ to vary from one to five and then from eight to 12 for relatively short and relatively long mean processing times respectively. The step size is one for both cases. The coefficient of variation for $F_{i,k}$ is kept constant at 0.5 in both cases. For the exponential distribution, the mean processing time is set similarly and the standard deviation is proportional to the mean by definition.

In the SOSD case, we use uniform and normal distributions (because exponential distribution is asymmetric). For the uniform distribution, we set $E_{F_{ik}}[\tilde{e}_i]$ at three and 10 for relatively short and relatively long mean processing times respectively. The coefficient of variation for $F_{i,k}$ is varied from 0.1 to 0.9 with a step size of 0.2 for both uniform and normal distributions.

In total, we have 12,500 combinations in each case: FOSD uniform, FOSD exponential, SOSD uniform, and SOSD normal.

6.1. FOSD Ambiguity

To examine the effect of ambiguity on the supplier's and buyer's profits (and hence preferences), we compute the percentage changes in their expected profit, with the expected profit at the lowest ambiguity level as the base case. A negative/positive percentage change therefore implies that the port becomes less/more attractive as ambiguity increases. Figures 5 illustrates the percentage changes in the supplier's and buyer's profits in the uniform case.[10]

Figure 5 corroborates our earlier analytical result (see Proposition 3) that both the supplier and the buyer prefer less ambiguity with regular shipments (which have relatively long buffer time to selling season) but both (in expectation) benefit from more ambiguity with more time-sensitive shipments (with relatively short buffer times). The impact of ambiguity is more significant for regular shipments than that
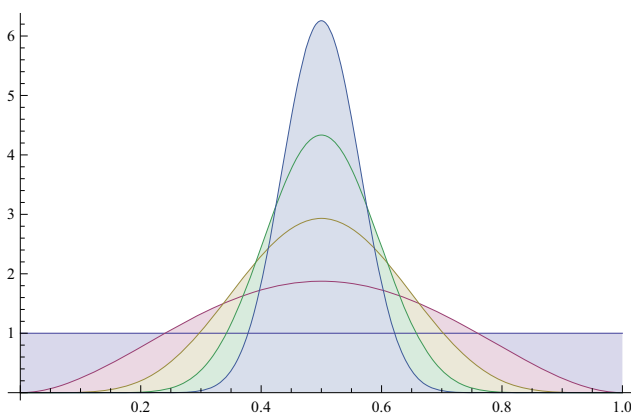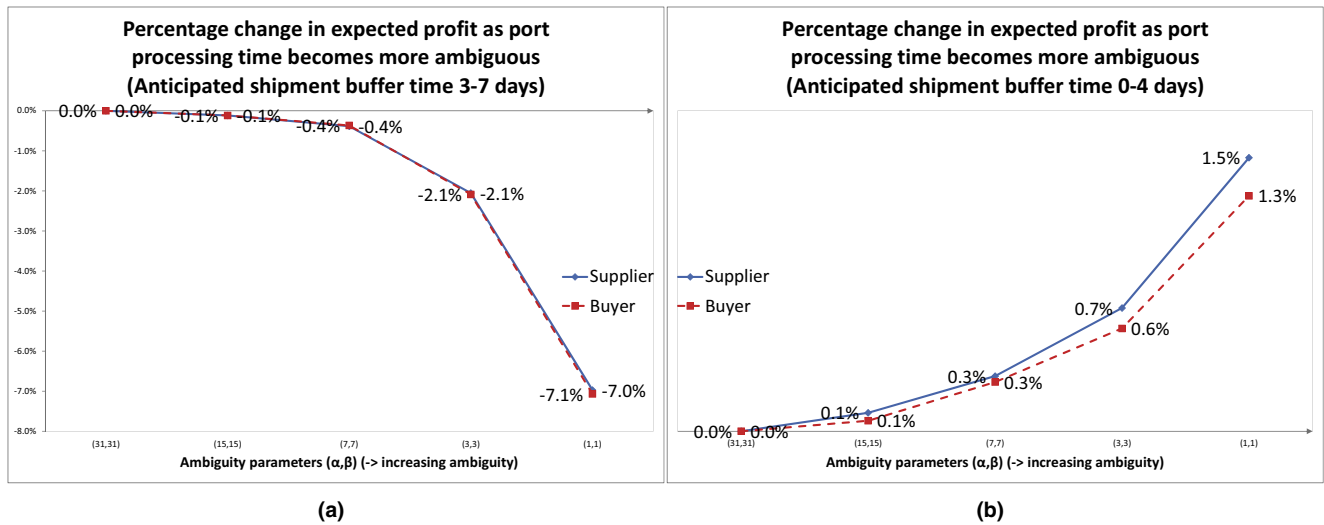
Figure 5   Illustration of the Effect of Port Processing Time Ambiguity (FOSD uniform) [Color figure can be viewed at wileyonlinelibrary.com]

(a)   (b)

for time-sensitive shipments: Compared with the least-ambiguity case, increased ambiguity reduces expected profit by as much as 7% for regular shipments but improves them by only up to 1.5% for time-sensitive shipments. In the exponential distribution case, we observe qualitatively similar results, that is, increased ambiguity reduces expected profit (by up to 4.4%) for regular shipments but slightly improves them (by up to 0.3%) for time-sensitive shipments.

While the above observations are related to Figure 3, they also hold more generally: out of 12,500 combinations, increased ambiguity reduces expected profit by up to 7.1%/4.4% (uniform/exponential) for regular shipments but improves expected profit by only up to 4.4%/1.8% (uniform/exponential) for time-sensitive shipments. However, for time-sensitive shipments, increased ambiguity improves expected profit by an *average* of only 0.59% under the exponential case, much less than the 2.14% increase in the uniform case. This suggests that if the ambiguity set consists of exponential distributions, both the supplier and the buyer are more likely to benefit from reduced ambiguity for regular shipments than from increased ambiguity for time-sensitive shipments.

As the ambiguity set $\mathcal{F}_i$ is enlarged, we observe that the impact of increased ambiguity is dampened regardless of whether the shipment is time-sensitive. For example, in the uniform case, while increased ambiguity reduces/improves expected profit by up to 11.3%/5.9% for regular/time-sensitive shipments when $|\mathcal{F}_i| = 3$, these are dampened to 5.3%/3.4% when $|\mathcal{F}_i| = 9$. Similarly, in the exponential case, increased ambiguity reduces/increases expected profit by up to 6.0%/2.3% for regular/time-sensitive shipments when $|\mathcal{F}_i| = 3$, and these are dampened to 3.5%/1.4% when $|\mathcal{F}_i| = 9$. Observe that the
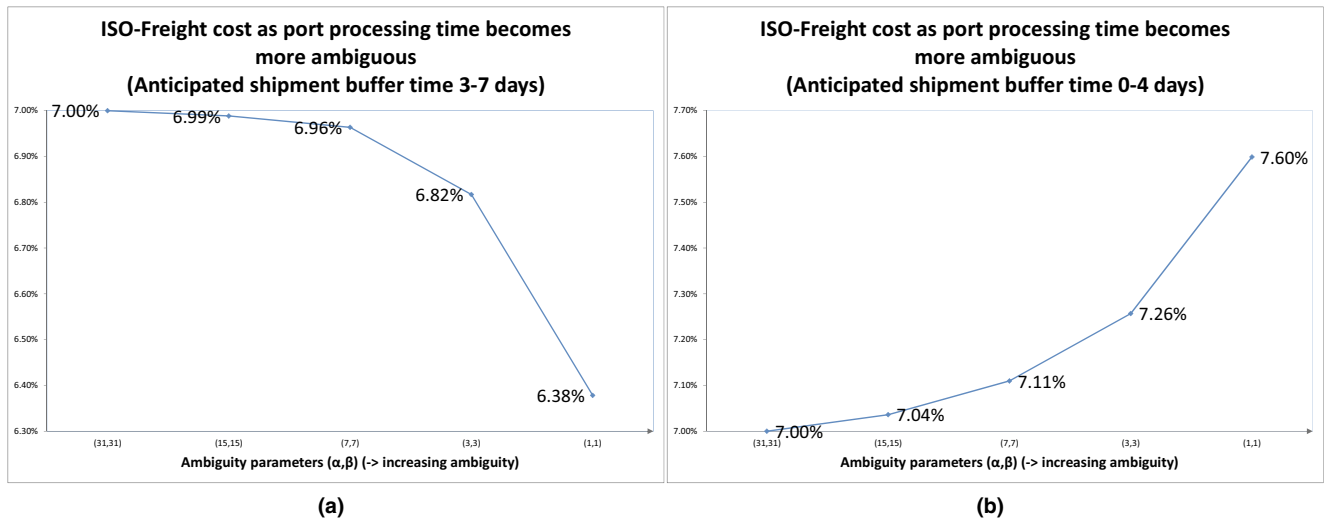
magnitude of the ambiguity impact between uniform and exponential cases is also dampened as $\mathcal{F}_i$ is enlarged. Because the distributional shape effect is weakened as $|\mathcal{F}_i|$ increases, the effect of the shipment buffer time manifests itself more clearly and the firms' preferences under greater ambiguity is therefore primarily driven by whether a shipment is time-sensitive: for regular shipments, they tend to select a port with less ambiguity (all else being equal), whereas ambiguous processing times are more palatable for time-sensitive shipments.

Nevertheless, reasonably accurate estimations for the three scenarios (e.g., optimistic, most likely, and pessimistic) enable firms to gain and utilize information on the shape of the distributions in $\mathcal{F}_i$ to make informed decisions. In particular, increased ambiguity is likely to have a larger impact on expected profit when the ambiguity set consists of uniform instead of exponential distributions, and reducing ambiguity is more likely to be a robust strategy with an exponential ambiguity set.

6.2. SOSD Ambiguity
We now turn our attention to the ambiguities manifested through SOSD. Similarly to the FOSD case, we compute the percentage changes in the buyer's and supplier's expected profits to deduce their preferences. We observe that ambiguity has only a negligible influence on firms' expected profits. In the uniform case, for example, increasing processing time ambiguity from the lowest to the highest level reduces the supplier's/buyer's expected profit by an average of 0.07%/0.07% for regular shipments and 0.52%/0.58% for time-sensitive shipments. Similarly, in the normal (distribution) case, the corresponding numbers are 0.01%/0.01% for regular shipments and

Figure 6   Illustration of ISO-Freight Cost Based on Buyer's Expected Profit (FOSD uniform) [Color figure can be viewed at wileyonlinelibrary.com]

(a)   (b)

1.10%/1.09% for time-sensitive shipments.[11] This suggests that ambiguities characterized by the SOSD have less of an impact on firms' preferences. For regular shipments with sufficient buffer times, the expected profits are virtually the same at various ambiguity levels, and firms often forgo transactions if shipment buffer times are excessively tight.

Part of the intuition is related to the fact that under an SOSD structure, elevated ambiguity levels do not necessarily correspond to a larger variance in the anticipated processing times. Coupled with the fact that anticipated processing times are kept constant, the result is that firms' expected profits are relatively insensitive (although decreasing slightly) to ambiguities in processing times. As such, the anticipated mean processing time becomes much more important (than in FOSD), because longer mean processing times can cause a transaction to become unprofitable to both the buyer and the supplier. An important managerial insight is that it is more important to reduce anticipated mean processing times if ambiguities arise through the SOSD structure.

With either FOSD- or SOSD-manifested ambiguity, the fraction of the supplier's transportation costs α_i has little impact on the effect of ambiguity. We observe no moderating effect of α_i on ambiguity levels. Part of the intuition is that the supplier fully prices transportation costs into the unit wholesale price w, such that whether the supplier bears a larger portion of transportation costs does not affect its and the buyer's expected profits. Thus, whether they adopt FOB or CIF Incoterms does not influence a port's attractiveness. This implies that the buyer need not insist on a particular port since its preference is aligned with that of the supplier.
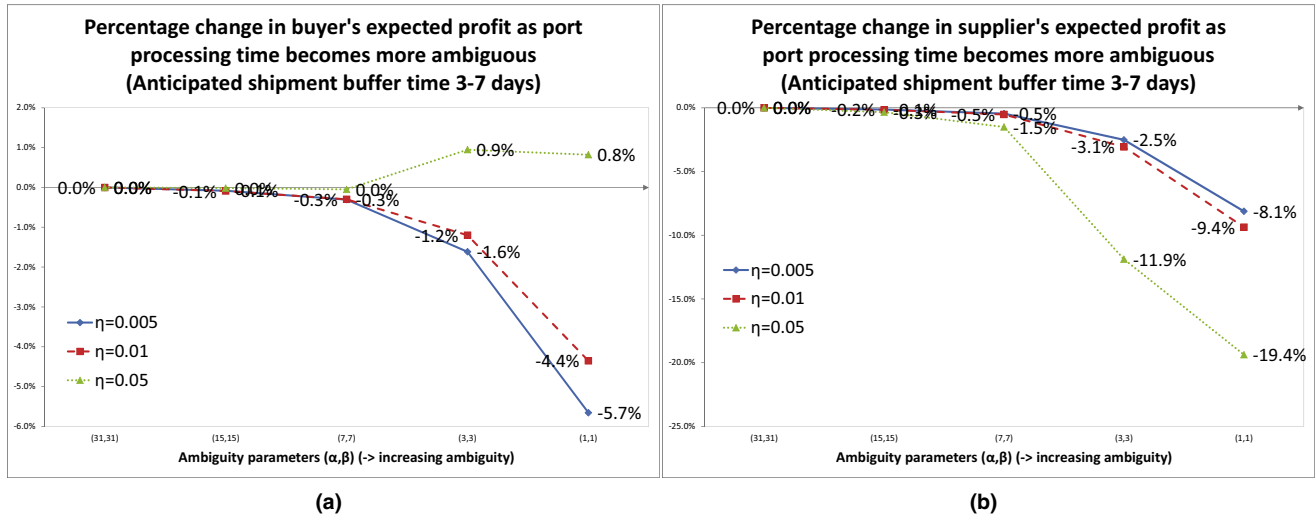
On the other hand, being aware of the nature of ambiguity is important to recognizing its effect on a port's attractiveness. When processing times are affected by discrete, traceable events (FOSD), reducing ambiguity enhances a port's attractiveness for regular shipments but not necessarily for time-sensitive shipments, all else being equal. In contrast, when processing times are affected by frequent, random noises (SOSD), reducing the mean processing time is paramount to enhancing a port's attractiveness. Likewise, a port's attractiveness is always enhanced with a shorter mean processing time, even under FOSD.

6.3. Freight Cost
The freight cost for a port can be influenced by many factors, such as overall distance, commodity size and weight, port efficiency, and fleet network. When selecting a port, the buyer must often therefore strike a balance between freight costs and anticipated processing times. Below we explore how large of an increase or decrease in the unit freight cost is required to compensate for increased processing time ambiguity. Specifically, we first calculate the buyer's expected profit at the unit freight cost of $7\% \times c$ with the least processing time ambiguity as a base case. We then increase processing time ambiguity and compute the corresponding unit freight costs that will maintain the buyer's expected profit. We assume that the fixed cost is absorbed in the unit freight cost, that is, $\phi_i = 0$.[12] Figure 6 illustrates the ensuing iso-freight cost curve.[13]

For regular shipments, a port with higher processing time ambiguity must compensate with a lower unit freight cost rate, and the reduction of this rate can be as much as -9% ($6.38\%/7\% - 1$) all else being equal. This is intuitive as a port with a less predictable

Figure 7 Illustration of the Effect of Ambiguity in Port Processing Time When the Buyer is Ambiguity-Averse (FOSD uniform, regular shipments) [Color figure can be viewed at wileyonlinelibrary.com]

(a)

(b)

processing time is not as attractive as one with a more consistent processing time, but reducing the unit freight rate can compensate for such processing time ambiguity. It is worth pointing out that freight costs often include both transport costs and handling costs at the port, many aspects of which are therefore beyond the control of the port. Achieving a 9% reduction requires either cost reductions by all parties or a higher (more than 9%) cost reduction by the port itself. On the other hand, improving processing time predictability enables a port to charge a higher freight rate and still maintain a competitive advantage.

For time-sensitive shipments, a port with a higher degree of processing time ambiguity can potentially charge a higher unit freight cost rate. This is surprising, but consistent with our earlier results that increased ambiguity can potentially benefit the buyer: a higher level of ambiguity implies that the port can process the shipment more quickly than the anticipated mean processing time with a non-negligible probability. This possibility is beneficial for time-sensitive shipments, especially when the unit production cost is low and the profit margin is high. In practice, however, this observation may not hold as a norm because the buyer can be ambiguity-averse.

6.4. Aversion to Ambiguity

If the buyer is ambiguity-averse, it is likely to increasingly prefer ports with less ambiguity—regardless of whether the shipment is time-sensitive. On the other hand, it is not immediately clear how ambiguity affects the supplier's preference. We slightly adapt the ambiguity aversion function developed in section 5.3 by using $\Theta(x) = 1 - e^{-\eta x}$ to calculate the buyer's and supplier's expected profits. We vary the coefficient of ambiguity $\eta = 0.005, 0.01, 0.05$. Recall that a
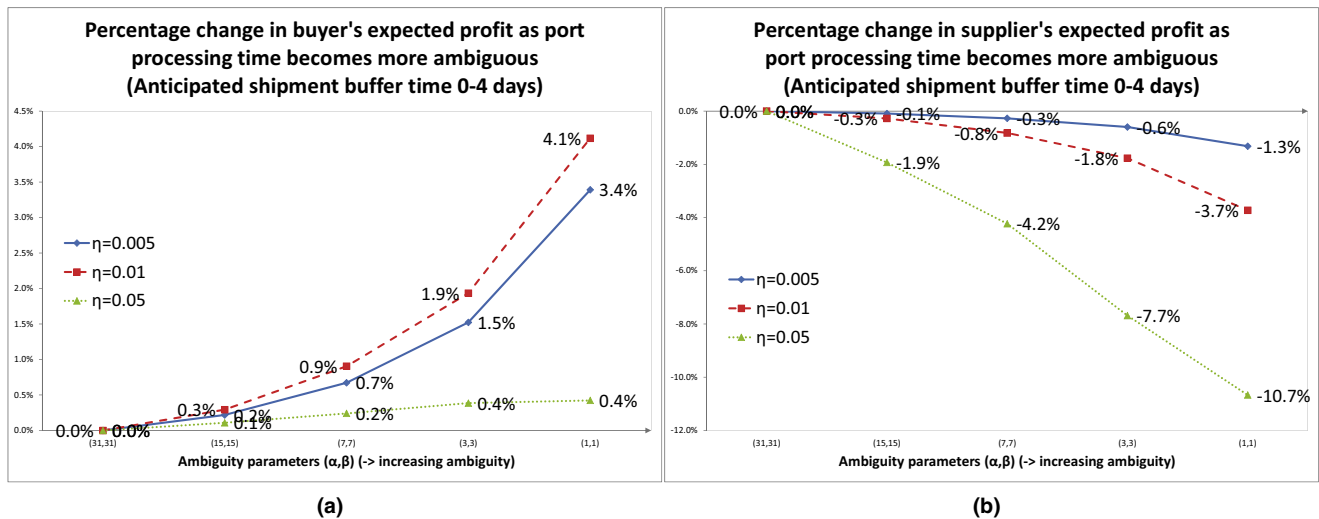
larger η corresponds to higher ambiguity aversion. Figures 7 and 8 illustrate the effect of ambiguity aversion for regular and time-sensitive shipments respectively. All other parameters used are identical to those in Figure 5. Note that with ambiguity aversion the expected profit is transformed by $\Theta(x) = 1 - e^{-\eta x}$, and hence the percentage changes in expected profit cannot be directly compared with those in Figure 5.

For regular shipments Figure 7a suggests that, contrary to our earlier intuition, an ambiguity-averse buyer may not be worse off with increased processing time ambiguity. In contrast, the supplier is always worse off. This is partly because ambiguity aversion leads the buyer to reduce its order quantity (Corollary 2), which in turn forces the supplier to quote lower prices. The burden of ambiguous processing times is therefore partially transferred from the buyer to the supplier (see Figure 7a and b at $\eta = 0.05$). The system as a whole is worse off: the buyer places a reduced order and the supplier earns less profit.

For time-sensitive shipments, increased ambiguity aversion curbs the buyer's preference toward ports with more ambiguous processing times. Figure 8a suggests that as the buyer becomes more ambiguity-averse (η increases) the attractiveness of ports with ambiguous processing times diminishes. Similar to the regular shipment case, Figure 8b suggests that the supplier is consistently worse off as the buyer becomes increasingly ambiguity-averse.

While the general observations in Figures 7 and 8 agree with our intuition that ambiguity aversion dampens the attractiveness of ports with ambiguous processing times for both regular and time-sensitive shipments, the impacts on the buyer and supplier are more nuanced. The buyer suffers less (or may even

Figure 8 Illustration of the Effect of Ambiguity in Port Processing Time When the Buyer is Ambiguity-Averse (FOSD uniform, time-sensitive shipments) [Color figure can be viewed at wileyonlinelibrary.com]

slightly benefit) from ambiguous processing times, which can transfer the burden of ambiguity to the supplier, whereas the supplier is consistently worse off. Because the system as a whole is worse off, however, the supplier is significantly better off if the ambiguity-averse buyer can select ports with less ambiguous processing times.

6.5. Fixed Wholesale Price

When the buyer is influential, the unit wholesale price is often fixed on a cost-plus basis for the supplier. It is therefore important to determine whether our earlier results continue to hold when the supplier quotes a fixed, cost-plus wholesale price. This is admittedly sub-optimal for the supplier but it eliminates double marginalization, which is inherent in most two-player supply chain models.

We find that our previous observations on the effect of ambiguity continue to hold with a fixed wholesale price. There are no qualitative differences in firms' preferences toward ambiguity with the exception of time-sensitive shipments under ambiguity manifested through an SOSD normal structure. Specifically, we observe that at a fixed price the buyer's expected profit increases with processing time ambiguity for time-sensitive shipments. The supplier's preference is *not* influenced by ambiguity levels because the buyer's optimal order quantity is not influenced by ambiguity. The fixed wholesale price scheme eliminates the inherent inefficiency of double marginalization and hence makes it profitable for the buyer to ship the product even if the shipment buffer time is very tight. We observe, however, that this is only possible when the unit production cost is relatively low (i.e., $c = 1.0$); at higher costs the system behaves as in the previous analysis with both firms

forgoing the transaction if the shipping buffer time is deemed too tight.

Thus under specific conditions, especially when the unit production cost is relatively low and the shipment is time-sensitive, the buyer's and supplier's preferences are no longer strictly aligned: the supplier is indifferent between choosing a port with high ambiguity or with low ambiguity whereas the buyer prefers a port with high ambiguity. In such cases, the buyer is better off specifying the port itself in the contract, as opposed to delegating the decision to the supplier.

Another difference we observe at a fixed wholesale price is that the supplier is consistently better off with higher values of α_i. This suggests that the supplier strictly prefers a CIF contract to an FOB contract at a fixed wholesale price. This is not the case when the supplier optimizes the wholesale price and transfers the transportation costs to the buyer. At a fixed wholesale price, the supplier earns an additional margin on the transportation costs and hence prefers CIF contracts, which incorporates these transportation costs. This observation is consistent with real practice where suppliers usually charge a premium for arranging transportation all the way to the buyer's destination.

As a result, if the total (origin to destination) transportation cost is similar but the supplier is closer to port A than to port B, then all else being equal the supplier is equally likely to choose either port at an optimal wholesale price (either with FOB or CIF) but will prefer port B at a fixed wholesale price (with CIF). Thus at a fixed wholesale price, the impact of ambiguity is moderated by a port's proximity to the supplier's facility under a CIF contract. In particular, a fixed wholesale price reduces the alignment

between the preferences of the supplier and buyer. For example, for regular shipments both the supplier and the buyer prefer a port with less ambiguity (port C), but, because another port (port D) is farther from the supplier's facility, the supplier may opt for port D despite port C's less ambiguous processing times.

7. Conclusion

Processing time predictability is an important factor influencing a port's competitiveness in global container transportation. Given that geographic locations are fixed, it is both theoretically interesting and practically useful to understand whether and how improved processing time predictability can enhance a port's attractiveness to shippers. We consider a situation where a firm does not know the true distribution of a port's processing time but relies on second-order probabilities to obtain a plausible set of processing time distributions to optimize its shipments.

While it is always desirable for a port to reduce average processing times, all else being equal, the associated costs can be prohibitive. On the other hand, improving predictability is often more cost-effective, as is the main thrust of this study. As an initial step to explore ambiguity associated with port processing times, we find that the effect of ambiguity is not straightforward.

We prove that an ambiguity-neutral firm should select a port with less ambiguity for regular shipments but can be better off opting for a port with more ambiguity for time-sensitive shipments. Such a preference is robust to differences in the firm's belief structure (ambiguity set) about port processing times, which holds regardless of whether the firm forms an ambiguity set through uniform or exponential distributions. The uniform distribution reflects situations where the firm can reasonably estimate the upper and lower bounds of the port processing times but is minimally informed about their distributional shapes, whereas an exponential distribution reflects situations where the firm knows the mean processing times but is unsure about their upper bounds. Despite these significant differences, the firm's preference for less/more ambiguity for regular/time-sensitive shipments continues to hold.

An important caveat, however, is that the distributional shape in the ambiguity set does influence the *magnitude* of the expected benefit. In particular, when port processing times are exponentially distributed (i.e., with long tails), a firm derives a greater benefit if it selects a port with less ambiguity for regular shipments than if it selects a port with more ambiguity for time-sensitive shipments. In contrast, with symmetric, bounded processing times (i.e., with a uniform

distribution), the firm can benefit equally from selecting a port with more ambiguity for time-sensitive shipments and from selecting a port with less ambiguity for regular shipments. The firm thus benefits more significantly from selecting a port with less ambiguity for regular shipments when port processing times exhibit long tails.

When the firm becomes less certain about the mean or lower/upper bounds of processing times (i.e., an enlarged ambiguity set), the impact of ambiguity on the expected profit dampens, as does the effect of the shape of the underlying distributions. The firm's preference is more saliently influenced by the time sensitivity of shipment: the firm prefers a port with less ambiguity for regular shipments but is better off with an ambiguous port for time-sensitive shipments. A port with erratic operations *may* enable a shipment to pass through more quickly than a port with more consistent operations given the same expected processing time.

If a firm is ambiguity-averse, however, a port with less ambiguity is increasingly preferred, especially by the supplier. Coupled with the fact that exponential distributions are often considered more realistic for estimating port processing times, it is safe to conclude that a port can indeed improve its competitiveness by improving its operational predictability.

Our aim is not to prescribe operational decision support but rather to gain an improved understanding of how ambiguity can affect a port's attractiveness in the global container industry. We hope that this study will pave the way for future work to further investigate relevant factors that can improve a port's competitiveness.

Acknowledgments

Notes

[1]Another factor that can influence port processing time is demurrage and detention stipulations, which can also influence carrier/forwarder preferences.
[2]The variability of average time at sea is primarily influenced by shipping routes.
[3]We have also conducted informal interviews with several senior shipping executives, and learned that it usually takes a container about one day longer

(expected time) to pass through terminals in Mainland China than through those in Hong Kong. The executives suggested, however, that the predictability of processing time and cost is their primary concern rather than expected time and is the main reason for preferring Hong Kong ports.

[4]Looks-like analysis is also referred to as analogous forecasting and a forecasting method for new products that exhibit similar traits to old products (Mas-Machuca et al. 2014).

[5]Caldeirinha et al. (2011), for example, note that port location itself is not a critical factor determining port performance and efficiency (p. 64).

[6]It is also methodologically more appealing when the second-order probability can be updated over time, such as the updating process proposed by Saghafian and Tomlin (2016). The definition of ambiguity, however, does not intrinsically rest on the time dimension, that is, the second-order probability need not be updated over time (see the verifiability issue discussed in Klibanoff et al. (2005, p. 1856). We ignore the mechanics of the updating process for the second-order probability to focus on the effect of ambiguity on port attractiveness. Nevertheless, our model still applies if there is an updating process—as long as the shipment decision needs to be made before the ambiguity is fully resolved.

[7]The generalized failure rate is defined as $xg(x)/\bar{G}(x)$. An IFR distribution always satisfies IGFR.

[8]Note that if the buyer is risk averse (but not necessarily ambiguity averse) then all else being equal the port with a more ambiguous processing time is less attractive.

[9]Here we use the unit production cost c as a baseline (as opposed to the unit selling price p_f or p_d) because freight costs are more commonly considered as a percentage of the cost of goods sold. In reality, freight costs vary significantly between carriers as fleet networks and hence transit times differ significantly. Lu et al. (2017) provides a more detailed discussion on carrier selection based on different transit times and freight costs.

[10]Figure 5a was obtained by setting $t = 17$, $l = 12$, $c = 3$, and $\alpha_i = 0.2$. The anticipated shipment buffer time is $T - l_i - E[\tilde{e}_i] = 20 - 12 - \{5 \sim 1\} = 3 \sim 7$. Figure 5b was obtained by setting $t = 17$, $l = 8$, $c = 1$, and $\alpha_i = 0.2$. The anticipated shipment buffer time is $T - l_i - E[\tilde{e}_i] = 20 - 8 - \{12 \sim 8\} = 0 \sim 4$.

[11]With time-sensitive shipments, however, firms choose to forgo transactions in 48.8% of the 12,500 observations. Moreover, unlike the FOSD, increased ambiguity typically reduces both the buyer's and supplier's profits regardless of whether the shipment is time-sensitive.

[12]Most commercial quotes are based on costs per 20- or 40-ft container (often with volume discounts), which suggests that fixed costs are somewhat absorbed into the quoted per-container cost.

[13]Figure 6 is obtained using the same set of parameter values as those in Figure 5.

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Supporting Information

Additional supporting information may be found online in the supporting information tab for this article:

Appendix S1: Proof.