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Selling Co-Products through a Distributor: The Impact on Product Line Design

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A vertical co-product technology simultaneously produces multiple outputs that differ along a rankable quality metric. Co-product manufacturers often sell products through a distributor. We examine a setting in which a manufacturer sells vertically differentiated co-products through a self-interested distributor to quality-sensitive end customers. The manufacturer determines its production, product line design, and wholesale prices. The distributor determines its purchase quantities and retail prices. In traditional product-line design, products can be produced independently of each other and higher-quality products have higher production costs. This literature established that the length of the product line (i.e., difference between highest and lowest qualities) is greater in an indirect channel than in a direct channel. By contrast, co-products cannot be produced independently of each other. Among other findings, we establish that this interdependency causes the opposite channel effect: for co-products, the length of the product line is smaller in an indirect channel than in a direct channel. Additionally, we show that there exists a theoretical contract, combining revenue sharing and reverse slotting fees, that eliminates the indirect channel distortions in both product line design and output quantities.

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1. Introduction

There are many important industries—commodities, semiconductors, and specialty materials, for example—in which production simultaneously generates multiple outputs. In some cases, the outputs ("co-products") serve different purposes; examples include certain agriproduct processes (Boyabatli 2015, Boyabatli et al. 2017), chemical processes (Chen et al. 2017), and oil refining (Dong et al. 2014), In other cases, the outputs serve the same basic purpose but differ in their quality levels along a dimension for which more is better. Examples include microprocessors that differ in speed, light-emitting diodes (LEDs) that differ in luminescence, and synthetic industrial diamonds that differ in strength (Chen et al. 2013, 2017). In this latter "vertically-differentiated" case, production typically results in a spectrum (i.e., a population distribution) of items from lower quality through to higher quality, and the manufacturer sorts and classifies the output into a set of grades. This sorting and classification is often referred to as "binning" in the semiconductor industries.

Pricing, production and product-line design—the number of grades to offer and their associated quality specifications—are key strategic decisions for vertical co-product manufacturers. A number of papers (Bansal and Transchel 2014, Chen et al. 2013, Chen et al. 2017, Min and Oren 1995, Tomlin and Wang 2008, Transchel et al. 2016) have explored certain aspects of these inter-related decisions in settings where the manufacturer sells directly to customers. An important reality absent from these papers, however, is that co-product manufacturers often sell their goods through an indirect channel; that is, they wholesale co-products to self-interested distributors who in turn sell them to end customers.

Cree, an LED manufacturer, originally used a direct sales strategy but in 2002 Cree gave Sumitomo exclusive distribution rights for its products in Japan (Annual Report 2003). By 2005, Cree was "actively negotiating distributor agreements to expand coverage in the United States, Europe, and Asia" (Annual Report 2006, front matter page 5). In 2010, Cree reported that "a substantial portion of our products are sold through distributors" (Annual Report 2010, page 6), and by 2017 56% of its overall revenue came from sales to distributors (Annual Report 2017).[1] The indirect channel is also meaningful for other co-product manufacturers. Intel's indirect channel "represents approximately 20% of its total CPU shipments" (Wieland et al. 2012, p. 517). Element Six—a leading industrial diamond producer—sells through authorized distributors, oftentimes with an exclusive distributor in a country.[2]

If a firm, such as Cree, considers moving in the direction of an indirect channel strategy, then it is vital that it understands whether and how this will influence its product line design and the channel profits. The answer to this question is not obvious because when selling through an indirect channel, the manufacturer needs to take into account the self-interested distributor's purchasing and retail-pricing strategies in designing its product line, setting wholesale prices, and choosing a production quantity. All else equal, double marginalization in the indirect channel will presumably distort the product line design, but in what manner, that is, will the manufacturer offer more or fewer products and how will the product qualities change? (If, as one might expect, the manufacturer reduces its overall production quantity due to double marginalization, will it increase or decrease output qualities? This question is nontrivial because, as discussed below, output quantities and qualities are interdependent for co-products. Therefore, the manufacturer might either reduce product qualities to compensate for the reduced production quantity or increase qualities to further reduce sales volume). Channel profit will be reduced but does there exist a distributor contract that eliminates this profit loss? These are the central questions examined in this study.

We analyze a model in which a co-product manufacturer sells its products through a distributor. The manufacturer determines its product line, its production quantity, and the wholesale prices to charge the distributor. The distributor in turn chooses its purchase quantities and the retail prices to charge the end market of quality-sensitive customers.[3] For any given production quantity, we characterize the equilibrium wholesale prices, distributor purchase quantities, and retail prices for any optimal product line design in this manufacturer-distributor game. Furthermore, we characterize how the optimal production quantity and product line design in this indirect channel compare to the optimal quantity and line decisions that the manufacturer would make if selling directly to end customers. Two key metrics in product line design are the number of products in the line and the difference in qualities between the highest and lowest quality products. We refer to these two metrics as the size and length of the product line respectively.[4] We establish that the co-product manufacturer should decrease the size and length of its product line when selling through an indirect channel as compared to a direct channel.

This length-reduction impact lies in stark contrast to what has been established in the traditional vertically differentiated product line design problem in which products can be produced independently of each other and higher-quality products have higher marginal production costs. We refer to this as the "independent-product" setting to distinguish it from co-products. For the independent product setting, Villas-Boas (1998) established that when selling through an indirect channel, "the best strategy for the manufacturer is to increase the differences in the products [qualities] in comparison to the direct selling/coordinated channel case" (p. 156). In other words, the length of the product line is greater in an indirect channel as compared to a direct channel. By definition, co-products cannot be produced independently of each other. Their supply quantities are (endogenously) proportionally related through the technology's output quality spectrum and the manufacturer's product quality (grade specification) choices. Our results reveal that this fundamental attribute of co-product technologies—that product supplies are interdependent—gives rise to the diametrically opposite effect that an indirect channel has on the product-line length in co-product settings as compared to independent product settings.

The remainder of this study is organized as follows. The most relevant literature is reviewed in section 2. The model is described in section 3. Results are presented and discussed in section 4. Several extensions are examined in section 5 and conclusions are presented in section 6. Some additional technical results are given in the Appendix. All proofs of results in the main paper and Appendix can be found in the Supplementary material (i.e., Appendix S1). This supplement also contains some additional material referenced in the main paper.

2. Literature Review

Our work is related to three different streams of literature: Vertically differentiated co-product operations, product line design, and product quality choice in an indirect selling channel.

Much of the early research on vertically differentiated co-products (e.g., Bitran and Dasu 1992, Bitran and Gilbert 1994, Bitran and Leong 1992, Gerchak et al. 1996, Hsu and Bassok 1999, Nahmias and Moinzadeh 1997) explored the operational decisions of production quantity and downward substitution, that is, downgrading a higher-quality product to satisfy the demand for a lower-quality product. A number of more-recent two-product papers also consider the firm's pricing question which influences the demand for each co-product (e.g., Bansal and Transchel 2014, Chen et al. 2017, Tomlin and Wang 2008). The co-product literature on pricing and product line design (Chen et al. 2013, Min and Oren 1995, Transchel et al. 2016) is the most related to our work. The above papers on product line design and/or pricing assume that the manufacturer sells co-products directly to end customers. To the best of our knowledge, ours is the first co-product paper to examine pricing and product line design in an indirect channel, that is, when the manufacturer sells co-products to a self-interested distributor who in turn prices and sells products in the end market. Our study is closely related to Chen et al. (2013) in that, as we will describe in section 3, our base model adopts its production and end-customer demand features from Chen et al. (2013). The primary difference in our base model is that we consider a manufacturer that sells to a distributor who in turn sells to end customers. As such, Chen et al. (2013) serves as the direct-channel benchmark for our indirect channel model. We also extend our analysis to consider alternative production cost structures not considered in Chen et al. (2013).

There is a large literature on product line design for vertically differentiated products that traces back to the seminal works of Mussa and Rosen (1978) and Moorthy (1984). The underlying paradigm in this literature is that utility-maximizing customers vary in their valuation of quality (but all agree that higher quality is better ignoring prices) and that the firm can choose to offer one or more products that differ in quality. Higher-quality products have a higher marginal production cost and products can be produced independently of each other. The firm—selling directly to customers—chooses the number of products to offer and the associated quality level for each offered product. Some papers also consider settings in which the number of products is fixed (e.g., Moorthy 1984). Netessine and Taylor (2007) explore the impact of operational costs (e.g., inventory) on product line design. Qi et al. (2016) examine the case when customers are heterogeneous in not only their marginal valuation for quality but also their reservation valuation for a basic product of minimal quality. The vertically differentiated assortment problem is closely related to product line design but the firm chooses from a predetermined set of quality levels (e.g., Pan and Honhon 2012).

Some of the horizontally differentiated assortment design literature has examined the impact of indirect selling channels. Aydin and Hausman (2009) study the retailer's assortment decision in a decentralized supply chain where the manufacturer's assortment is exogenously given. They find that the retailer chooses a (weakly) smaller assortment in the decentralized supply chain than in the centralized supply chain. Liu and Cui (2010) investigate whether or not the manufacturer would extend its product line, that is, introduce an additional product, and find that the manufacturer has more incentives to do so in an indirect channel than in a direct channel. In a two-product setting with demand uncertainty, Dong et al. (2018) examine the impact of push and pull contracts on the manufacturer's assortment choice, that is, whether to offer one or two products.

Closer to our work is Villas-Boas (1998) who studies a manufacturer that sells two vertically differentiated products to two market segments through a downstream retailer. He establishes that the product line is more differentiated (i.e., a greater difference in quality levels between the high-end and low-end products) in an indirect channel as compared to a direct channel in which the manufacturer sells directly to the end market segments. In an indirect channel, the manufacturer maintains the same quality for the high-end product but reduces the quality of the low-end product. This finding is extended to the case of a continuum of customer types and an infinite number of products in the unabridged version (Villas-Boas 1996). In this study, we will establish that the indirect channel has the exact opposite effect in a co-product setting: a co-product manufacturer will offer a less differentiated product line in an indirect channel than in a direct channel.

Finally, we note that our paper is somewhat related to the single-product literature that explores the impact of an indirect channel on the product's quality. Jeuland and Shugan (1983) and Economides (1999) find that, due to a lower profit margin in the indirect channel, the manufacturer reduces the product's quality as compared to a direct channel. Xu (2009) endogenizes the pricing decision in a more general setting and shows that the impact of the indirect channel depends on the shape of the marginal revenue function. Shi et al. (2013) considers a setting with two dimensions of customer heterogeneity. Recently, Jerath et al. (2017) investigate the impact of demand uncertainty and inventory risk allocation in the indirect channel. Ha et al. (2015) explores whether a supplier who sells a single product through a retailer should also sell a single product (of a possibly different quality) in a direct channel. They find that if the supplier encroaches (sells directly) then it sells a weakly higher quality product in the direct channel.

3. The Model

We model a co-product manufacturer that sells to a distributor who in turn sells to end customers. As is common in the product-line design literature (e.g., Chen et al. 2013, Moorthy 1984, Pan and Honhon 2012, Villas-Boas 1998), we consider a single period model. In what follows, we define the manufacturer, distributor and end-customer components of the model before describing the manufacturer-distributor game.

Manufacturer: We adopt a similar production and classification model to Chen et al. (2013). The manufacturer operates a vertical co-product technology that generates a deterministic output quality spectrum defined by the distribution function $F(x)$, where $F(x)$ is continuous and increasing with positive support $[\underline{x}, \bar{x}]$ so that $F(\underline{x}) = 0$ and $F(\bar{x}) = 1$. Let $\bar{F}(x) = 1 - F(x)$. The proportion of the output whose quality levels lie between some x_i and x_j, where $\underline{x} \le x_i < x_j \le \bar{x}$, is given by $F(x_j) - F(x_i)$. A production quantity of Q therefore yields a quantity $Q[F(x_j) - F(x_i)]$ of products whose qualities lie in the range $[x_i, x_j)$. The manufacturer's cost of producing a quantity Q is denoted by $C_p(Q)$, where $C_p(Q)$ is assumed to be (weakly) convex increasing and twice differentiable. We extend our analysis to a concave production cost in section 5.2.

After production, the manufacturer classifies the output. That is, it evaluates the quality of each (infinitesimal) unit and sorts the output into N different quality grades, where grade n corresponds to the quality interval $[x_n, x_{n+1})$ for $n = 1, 2, \ldots, N$ and we define $x_{N+1} = \bar{x}$. Without loss of generality, we assume that outputs with quality levels in $[\underline{x}, x_1)$ are discarded. The classification (or "binning") cost increases in the number of units Q to be evaluated and the number of grades N used. It is given by $C_b(Q, N) = b_0 + b_1 Q + b_2(N - 1)$. Alternative classification cost structures are considered in section 5.2.

The manufacturer determines the production quantity Q and the product line design, that is, the number of grades N and the grade specification vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_N)$. This results in a quantity available of $a_n = Q[F(x_{n+1}) - F(x_n)]$ for grade $n = 1, \ldots, N$.[5] For each grade n, the manufacturer sets a per-unit wholesale price w_n that it charges the distributor. To rule out the uninteresting case in which the manufacturer does not classify the output and simply offers a single grade of quality interval $[\underline{x}, \bar{x})$, we assume the classification-related costs b_0 and b_1 are not so large as to render classification uneconomical. Moreover, we rule out other trivial cases by assuming throughout the paper that it is always profitable for the manufacturer to produce a positive quantity of at least one grade, that is, $Q > 0$ and $x_1 < \bar{x}$.

Distributor: The distributor purchases products from the manufacturer and sells to end customers. For each grade $n = 1, \ldots, N$, the distributor determines the quantity q_n to purchase from the manufacturer and the retail price p_n charged to end customers. Similar to Villas-Boas (1996, 1998), the only cost incurred by the distributor is the purchase cost it pays to the manufacturer.

End customers: We adopt a standard vertically differentiated product line design customer model (e.g., Bhargava and Choudhary 2001, Pan and Honhon 2012, Villas-Boas 1996). There is a continuum of infinitesimal customers and the market size is normalized to one. Customers are heterogeneous in their valuation of quality, with a customer of type θ deriving a utility of $\theta x - p$ for buying a product of quality x at a price p. The population valuation distribution for θ is Uniform within [0, 1].[6] Neither firm (manufacturer or distributor) can directly observe an individual customer's type. Define $x_0 = p_0 = 0$ as representing the no-purchase option (which has zero utility). Thus, there are $N + 1$ choices for customers, and grade n is associated with a quality level x_n and a retail price p_n where $n = 0, 1, 2, \ldots, N$.[7] Customers simultaneously decide which grade to buy in order to maximize their own utility. In the event of a customer's first-choice product being unavailable, we assume that the customer either spills down to their next-preferred lower-quality grade or the firm fills the demand by providing a higher-quality grade at the original grade's price, that is, downward substitution. (As will be shown in Lemma 1, no downward substitution/spill-down occurs in the optimal solution to the distributor's problem. Hence, all our results continue to hold even when downward substitution and spill-down are not allowed).

The direct channel benchmark: There is no distributor in a direct channel setting and the manufacturer sells directly to the end customers. The manufacturer determines the production quantity Q, the product line design N and $\boldsymbol{x}$, and the retail prices $\boldsymbol{p} = (p_1, p_2, \ldots, p_N)$. This is the setting examined in Chen et al. (2013), and we will avail of a number of their results when comparing the product line design in an indirect channel to that of a direct channel.

The indirect channel manufacturer-distributor game: We model the indirect channel setting as a two-stage Stackelberg game. In the first stage, the manufacturer, as the Stackelberg leader, determines the production quantity Q, the product line design N and $\boldsymbol{x}$, and the wholesale prices $\boldsymbol{w} = (w_1, w_2, \ldots, w_N)$ so as to maximize its profit. We examine a more complex contract structure in section 5.3. In the second stage, after the manufacturer announces the quality levels $\boldsymbol{x}$, the wholesale prices $\boldsymbol{w}$, and the quantities available $\boldsymbol{a} = (a_1, a_2, \ldots, a_N)$ for each grade $n = 1, \ldots, N$, the distributor determines the quantity of each grade $\boldsymbol{q} = (q_1, q_2, \ldots, q_N)$ to purchase from the manufacturer and the retail prices $\boldsymbol{p}$ to charge end customers so as to maximize its profit. Let $d_n(\boldsymbol{x}, \boldsymbol{p})$ be the end-customer first-choice demand for grade n, which depends on both $\boldsymbol{x}$ and $\boldsymbol{p}$.

In formulating the distributor's purchase quantity and pricing problem, it is helpful to use the concept of echelon purchase quantities and echelon demands. Define $d_n^E(\boldsymbol{x}, \boldsymbol{p}) = \sum_{i=n}^N d_i(\boldsymbol{x}, \boldsymbol{p})$ as the echelon demand for grade n, that is, the total demand for grades n and higher. Likewise, define $q_n^E = \sum_{i=n}^N q_i$ as the echelon purchase quantity for grade n, and let $\boldsymbol{q}^E = (q_1^E, q_2^E, \ldots, q_N^E)$. We note that $q_n = q_n^E - q_{n+1}^E$ for $n = 1, \ldots, N-1$ and $q_N = q_N^E$. Under the assumption of downward substitution/spill-down, the echelon sales quantity for grade n, that is, sales of grades $n, n+1, \ldots, N$, is given by $s_n^E(\boldsymbol{x}, \boldsymbol{p}, q_n^E) = \min(d_n^E(\boldsymbol{x}, \boldsymbol{p}), q_n^E)$. For any given manufacturers-specified quality levels $\boldsymbol{x}$, wholesale prices $\boldsymbol{w}$, and quantities available $\boldsymbol{a}$, the distributor's purchase quantity and pricing problem can then be written as follows.

$$\begin{aligned}\max_{\boldsymbol{p},\boldsymbol{q}^E}\ & \sum_{n=1}^N (p_n - p_{n-1}) \min(d_n^E(\boldsymbol{x}, \boldsymbol{p}), q_n^E) \\ & - \sum_{n=1}^N (w_n - w_{n-1}) q_n^E \qquad (1)\\ \text{s.t. }\ & 0 \le q_n^E - q_{n+1}^E \le a_n \text{ for } n = 1, 2, \ldots, N-1 \\ & 0 \le q_N^E \le a_N,\end{aligned}$$

where the objective function is the overall sales revenue minus the overall purchase cost, the constraints reflect that the purchase quantity of grade $n = 1, \ldots, N$ cannot exceed the quantity available, and we define $w_0 = 0$ for notational convenience.

Following the standard technique in the vertical differentiation literature (e.g., Chen et al. 2013, Pan and Honhon 2012), we can transform the distributor's pricing decision to determining a set of quality valuation cutoff points $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_N)$ such that $0 \le \theta_1 \le \cdots \le \theta_N \le 1$ and a customer with valuation $\theta \in [\theta_n, \theta_{n+1})$ prefers to buy grade-n. Defining $\theta_{N+1} = 1$ and $\theta_0 = 0$ for convenience, there is a unique relationship between the cutoffs and prices given by $\theta_n = \frac{p_n - p_{n-1}}{x_n - x_{n-1}}$ for $n = 1, 2, \ldots, N$. The demand for grade n is given by $d_n(\boldsymbol{x}, \boldsymbol{p}) = \theta_{n+1} - \theta_n$. It then follows that echelon demand for grade n is given by $1 - \theta_n$. The distributor's grade pricing and echelon purchase quantity problem (1) can then be reformulated as the following valuation cutoff and echelon purchase quantity problem:

DISTRIBUTOR'S PROBLEM

$$\begin{aligned}\max_{\boldsymbol{\theta},\boldsymbol{q}}\ & \sum_{n=1}^N \theta_n (x_n - x_{n-1}) \min(1 - \theta_n, q_n^E) \\ & - \sum_{n=1}^N (w_n - w_{n-1}) q_n^E \qquad (2)\\ \text{s.t. }\ & 0 \le \theta_1 \le \theta_2 \le \cdots \le \theta_N \le 1 \\ & 0 \le q_n^E - q_{n+1}^E \le a_n \text{ for } n = 1, 2, \ldots, N-1 \\ & 0 \le q_N^E \le a_N.\end{aligned}$$

The manufacturer takes into account the distributor's profit maximizing behavior when choosing its production quantity Q, product line design N and $\boldsymbol{x}$, and wholesale prices $\boldsymbol{w} = (w_1, w_2, \ldots, w_N)$. The manufacturer's problem can be written as:

MANUFACTURER'S PROBLEM

$$\max_{\boldsymbol{x},\boldsymbol{w},N,Q}\ \sum_{n=1}^N (w_n - w_{n-1}) q_n^E - C_b(Q, N) - C_p(Q) \qquad (3a)$$

$$\text{s.t. } \boldsymbol{q}^E \text{ is the optimal solution to Distributor Problem (2)} \qquad (3b)$$

$$\underline{x} \le x_1 \le x_2, \ldots, \le x_N \le \bar{x} \qquad (3c)$$

$$a_n = Q[F(x_{n+1}) - F(x_n)] \text{ for } n = 1, 2, \ldots, N \qquad (3d)$$

$$Q \ge 0, N \text{ is a positive integer,} \qquad (3e)$$

where the objective function is the overall revenue received from distributor purchases minus the production and classification cost, constraint (3b) reflects the distributor's optimal echelon purchase quantities in response to the manufacturers decisions, and the other constraints respectively reflect the weakly increasing grade qualities, the quantity available expression, that production is nonnegative and there must be a positive integer number of grades. We will use the terms increase and decrease in their weak sense throughout the rest of this study.

We close this section by noting that our problem is not amenable to the backward induction approach commonly adopted to find the equilibrium in Stackelberg games. In backward induction, one has to explicitly derive the distributor's best response to any possible choice of Q, N, $\boldsymbol{x}$ and $\boldsymbol{w}$. Our challenge is that the number of grades N can take on any arbitrary positive number and, moreover, even for a given N, depending on the vectors $\boldsymbol{x}$ and $\boldsymbol{w}$, the distributor might only purchase a strict subset of the co-products and this possibility leads to an inordinate number of subcases to be examined. Instead of backward induction, we analyze our two-stage game by directly solving problem (3) as a bilevel optimization program in which the manufacturer maximizes its profit at the upper level while anticipating the distributor's optimal reactions in the lower-level problem (2). The optimal solution to problem (3) together with the distributor's optimal echelon purchase quantities constitutes the equilibrium in the Stackelberg game. Interested readers are referred to Colson et al. (2007) for an overview of bilevel optimization. Solving bilevel programs often requires numerical methods even when each player's problem is convex (Bard 1988). However, we are able to leverage structural properties of our bilevel program to substantially simplify our problem by eliminating many impossible equilibrium outcomes.

4. The Results

4.1. Preliminaries: Purchase Quantities and Wholesale and Retail Prices

Before exploring the impact of the indirect channel on the manufacturer's product line design and profit, we first develop some important properties of the manufacturer-distributor bi-level program that will allow us to characterize (for any given manufacturer production quantity) the relationship between the manufacturer's optimal grade specification vector, the manufacturer's optimal wholesale prices and the distributor's optimal retail prices. That in turn will allow us to reduce the manufacturer's overall problem to one of choosing a production quantity Q and a product line design, that is, the number of grades N and grade specification vector $\boldsymbol{x}$.

Recall that the echelon demand for grade n, that is, the demand for grades n and higher, is given by $1 - \theta_n$. One can therefore think of the distributor's problem in (2) as choosing the echelon purchase quantities and the echelon demands. The following lemma establishes that the echelon purchase quantity must exactly equal the echelon demand for each grade in any optimal solution to the distributor's problem.

LEMMA 1. *For any given manufacturer choice of production quantity Q, grade specification N and $\boldsymbol{x}$, and wholesale prices $\boldsymbol{w}$, the distributor's optimal cutoffs θ_n^* and echelon procurement quantities q_n^{E*} in problem* (2) *must satisfy $q_n^{E*} = 1 - \theta_n^*$ for $n = 1, 2, \ldots, N$.*

We can therefore replace the echelon purchase quantity q_n^E with the echelon demand $1 - \theta_n$ in both the distributor's and the manufacturer's problems (2) and (3), respectively. The distributor's problem (2) is then reduced to the following convex optimization problem:

$$\max_{\boldsymbol{\theta}} \quad \sum_{n=1}^{N} [\theta_n(x_n - x_{n-1}) - (w_n - w_{n-1})](1 - \theta_n) \quad (4a)$$

$$\text{s.t.} \quad 0 \le \theta_1 \le \theta_2 \le \cdots \le \theta_N \le 1 \quad (4b)$$

$$\theta_{n+1} - \theta_n \le a_n \text{ for } n = 1, 2, \ldots, N \quad (4c)$$

As discussed above, the solution to the distributor's problem is a constraint—see (3b) above—in the manufacturer's problem. Because problem (4) is convex, the distributor solution constraint (3b) in the manufacturer's problem can be characterized by the Karush–Kuhn–Tucker (KKT) conditions for problem (4).[8] We can thus replace (3b) with these KKT conditions, thereby reformulating the bilevel program as the following single-level optimization problem (Bard 1988):

$$\max_{\boldsymbol{\theta},\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{x},\boldsymbol{w},N,Q} \sum_{n=1}^{N} (w_n - w_{n-1})(1 - \theta_n) - C_b(Q, N) - C_p(Q) \quad (5a)$$

$$\text{s.t.} \quad 0 = (1 - 2\theta_n)(x_n - x_{n-1}) + w_n - w_{n-1} + \lambda_{n-1} - \lambda_n - \mu_{n-1} + \mu_n \text{ for } n = 2, 3, \ldots, N \quad (5b)$$

$$0 = (1 - 2\theta_1)x_1 + w_1 + \lambda_0 - \lambda_1 + \mu_1 \quad (5c)$$

$$0 = \lambda_n(\theta_{n+1} - \theta_n) \text{ for } n = 0, 1, 2, \ldots, N \quad (5d)$$

$$0 = \mu_n(a_n - \theta_{n+1} + \theta_n) \text{ for } n = 1, 2, \ldots, N \quad (5e)$$

$$\lambda_n \ge 0 \text{ for } n = 0, 1, \ldots, N, \mu_n \ge 0 \text{ for } n = 1, 2, \ldots, N \quad (5f)$$

$$0 \le \theta_1 \le \theta_2 \le \cdots \le \theta_N \le 1 \quad (5g)$$

$$\theta_{n+1} - \theta_n \le a_n \text{ for } n = 1, 2, \ldots, N \quad (5h)$$

$$\underline{x} \le x_1 \le x_2 \le \cdots \le x_N \le \bar{x} \quad (5i)$$

$$a_n = Q[F(x_{n+1}) - F(x_n)] \text{ for } n = 1, 2, \ldots, N \quad (5j)$$

$$Q \ge 0, N \text{ is a positive integer.} \quad (5k)$$

In the above formulation, $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are the Lagrangian multipliers associated with the inequality constraints (4b) and (4c) in the distributor's problem (4). The constraints (5b) and (5c) stem from the KKT conditions for problem (4). The constraints (5d) and (5e) that come from the complementary slackness conditions for problem (4) are not convex. However, these Lagrangian multipliers can be eliminated by exploiting the structure of problem (5). We relegate the analysis and several supplemental results for the above optimization problem to the Appendix.

We are now in a position to characterize (for any given manufacturer production quantity) the relationship between the manufacturer's optimal grade specification vector, the manufacturer's optimal wholesale prices and the distributors' optimal retail prices.

LEMMA 2. *For any given production quantity Q and optimal grade specification N^* and $\boldsymbol{x}^*$, the distributor's optimal cutoffs θ_n^* (and hence retail prices p_n^*) and the manufacturer's optimal wholesale prices w_n^* are given as follows:*

(*i*) $\theta_n^* = 1 - Q\bar{F}(x_n^*)$ *and* $p_n^* = \sum_{k=1}^{n} (x_k^* - x_{k-1}^*)(1 - Q\bar{F}(x_k^*))$ *for all* $n = 1, 2, \ldots, N^*$;

(*ii*) $w_n^* = \sum_{k=1}^{n} (x_k^* - x_{k-1}^*)(1 - 2Q\bar{F}(x_k^*))$ *for all* $n = 1, 2, \ldots, N^*$.

Leveraging Lemma 2, we can express the manufacturer's optimal revenue for any given production quantity Q and grade specification N and $\boldsymbol{x}$ as

$$R^I(Q, N, \boldsymbol{x}) = \sum_{n=1}^{N} (1 - 2Q\bar{F}(x_n))(x_n - x_{n-1})Q\bar{F}(x_n). \tag{6}$$

We can now reduce the manufacturer's problem to choosing a production quantity and grade specification vector as follows:

$$\max_{Q,N,\boldsymbol{x}} \quad R^I(Q, N, \boldsymbol{x}) - C_b(Q, N) - C_p(Q) \tag{7a}$$

$$\text{s.t.} \quad \underline{x} \le x_1 \le x_2 \le \cdots \le x_N \le \bar{x}, \tag{7b}$$

where the revenue function $R^I(Q, N, \boldsymbol{x})$ reflects the optimal wholesale prices (set by the manufacturer) and the corresponding optimal purchase quantities and retail prices (set by the distributor). In what follows we first explore the impact of the indirect channel on the product line design and then explore its effect on profits.

4.2. Impact of the Indirect Channel on the Product Line Design

A product line design is fully defined by the number of grades N and the grade specification (quality) vector $\boldsymbol{x}$. We are interested in the effect that selling through a distributor has on the manufacturer's optimal product line design. Two key metrics in the product line design literature are the length and size of a product line, where length is defined as the difference in quality between the highest and the lowest quality products (grades), that is, $x_N - x_1$, and size refers to the number of products (grades), that is, N. As we discussed in section 1, for the traditional product line design problem in which products can be produced independently of each other and higher-quality products have higher marginal production costs, Villas-Boas (1996, 1998) showed that selling through a distributor causes the manufacturer to increase the length and possibly decrease the size of its product line as compared to a direct channel. To preview our results, we will show that in a co-product setting the indirect channel has a diametrically opposite effect on product-line length; decreasing rather than increasing the length. We will also show that the indirect channel decreases the size of the product line but for a different reason than in the independent product setting.

We start by noting that it follows from Lemma 2 that the retailer earns a margin of $p_n^* - w_n^* = \sum_{k=1}^{n} (x_k^* - x_{k-1}^*)Q\bar{F}(x_k^*)$ for selling a unit of grade n. It can be shown that $x_n^* > x_{n-1}^*$ for all n in the manufacturer's product line design (see Proposition A3 in the Appendix); and this implies that $p_n^* - w_n^* > 0$, that is, the distributor earns a positive margin for each and every grade. The manufacturer therefore collects only a portion of the end-customer revenue for each grade but it incurs the full production and classification costs. As we will see, this double marginalization will have important effects on the manufacturer's production quantity and product line design.

Let us first focus on the impact of an indirect channel on the manufacturer's production quantity for a fixed product line design. We will use the superscript I to denote the indirect channel setting and the superscript D to denote the direct channel setting in all that follows. As formalized by the following proposition, double marginalization in an indirect channel has the effect of reducing the manufacturer's production quantity as compared to a direct channel.

PROPOSITION 1. *For any fixed product line design* (*i.e., number of grades N and grade specification vector $\boldsymbol{x}$*), *the manufacturer's optimal production quantities in the indirect and direct channel settings, Q^{*I} and Q^{*D} respectively, satisfy* (*i*) $Q^{*I} = \frac{1}{2}Q^{*D}$ *when the production cost $C_p(Q)$ is linear, and* (*ii*) $\frac{1}{2}Q^{*D} < Q^{*I} < Q^{*D}$ *when the production cost $C_p(Q)$ is strictly convex.*

The indirect channel production quantity is exactly half the direct channel production quantity if the production cost is linear in the quantity produced but is greater than half the direct channel production quantity if the production cost is convex. The production quantity reduction is a consequence of double marginalization in the indirect channel: the manufacturer's marginal revenue associated with increasing production is lower in the indirect channel than in the direct channel because it does not capture the full revenue associated with end-customer purchases. The marginal production cost is constant (increasing) in the quantity Q for a linear (convex) production cost but the marginal-revenue reduction (due to double marginalization) at any Q is unaffected by the production cost function. Therefore, the quantity distortion is greater when the production cost is linear. We emphasize that these relationships hold for any fixed product line design.

We now turn our attention to the product line design question, assuming for the moment that the size of the product line N is identical in the direct and indirect channels. Product line design is therefore the quality choice x_n for each grade 1, ..., N. Product line design in a co-product setting cannot be isolated from the production quantity decision because the supply of each grade is directly related to the production quantity Q and the quality output distribution $F(x)$. For example, a reduction in the production quantity, while holding the product line (grade specification vector) fixed, strictly decreases the echelon quantities of all grades. In particular, reducing production decreases both the overall product supply, that is, the echelon quantity of product 1, that is, $Q\bar{F}(x_1)$, and the supply of the highest quality product, that is, $Q\bar{F}(x_N)$. Ignoring price implications for the moment, the manufacturer could compensate (at least to some degree) for a production-induced supply decrease by (i) reducing the quality of the lowest grade x_1 to increase the overall product supply, and (ii) reducing the quality of the highest grade x_N to increase its supply. Recalling from Proposition 1 that double marginalization leads to a lower production quantity in an indirect channel than in a direct channel, one might then reasonably anticipate that the qualities of the lowest and highest quality grades would be lower in an indirect channel than in a direct channel to compensate for the reduced production.

As we now show, however, when prices are taken into account there is a counter-balancing force that pushes the grade quality choices in the opposite direction to this production quantity reduction force. For a given production quantity and grade specification vector, the manufacturer determines (through the wholesale prices) the end-customer sales volume of each grade. It sets the wholesale prices so that sales volumes equal the quantities available; see Lemmas 1 and 2 above. In particular, the manufacturer ensures that the sales volume of the highest grade N exactly equals its supply $Q\bar{F}(x_N)$. It can control this sales volume (for a given production quantity) by adjusting the grade quality x_N. Because of double marginalization, the manufacturer does not capture the full revenue associated with end-customer sales and, therefore, the manufacturer's marginal revenue associated with increasing the highest grade sales volume is lower in the indirect channel than in the direct channel. This means the manufacturer has an incentive to reduce the sales volume of the highest grade in the indirect channel as compared to the direct channel.[9] In other words, all else equal (production quantity and qualities of other grades), the manufacturer should set a higher quality for the highest grade in an indirect channel as compared to a direct channel because this reduces the sales volume of the highest grade. This is formalized in the following proposition.

Proposition 2. *If Q, N and $x_1, x_2, x_3, \ldots, x_{N-1}$ are fixed for both channels, the optimal quality of the highest grade is higher in the indirect channel than in the direct channel, that is, $x_N^{*I} \geq x_N^{*D}$.*

An analogous sales–volume reduction force should apply to the overall volume sold, that is, $Q\bar{F}(x_1)$, in a manner that puts upward pressure on the quality of the lowest grade.[10]

We therefore have two competing forces in the indirect channel. The production-quantity reduction force puts downward pressure on the qualities of the lowest and highest grades but the sales–volume reduction force puts upward pressure on these qualities. What is the net impact of these two forces? To answer this, we start by considering the linear production cost case. Recall that for the moment we are assuming the product line size N is identical in both the direct and indirect settings.

Proposition 3. *Assuming a linear production cost, then for any given N (fixed for both channels) if Q^{*D} and $\boldsymbol{x}^{*D}$ maximize the manufacturer's profit in the direct channel, then $Q^{*I} = \frac{1}{2}Q^{*D}$ and $\boldsymbol{x}^{*I} = \boldsymbol{x}^{*D}$ maximize the manufacturer's profit in the indirect channel.*

In other words, holding the product line size constant across channels, there exists an identical product line design that is optimal in both channels (though we do not assert uniqueness in this result). Therefore, in the special case of a linear production cost, the competing quality forces—production-quantity reduction and sales–volume reduction—cancel each other out and the product line is identical in the direct and indirect channel settings. When the production cost is convex, the production-quantity distortion is less severe than in the linear case (i.e., production reduces by less than 50%), with the result that the production-quantity downward pressure on quality is less strong and so the optimal product line design can differ in the indirect channel. To show this, we focus on two special case strategies for which the optimal product line size is unaffected by the channel: separation in which the manufacturer offers exactly one grade and complete classification in which the manufacturer offers an infinite number of grades. We remind the reader that, as described in section 3, the output classification (or "binning") cost is $C_b(Q, N) = b_0 + b_1 Q + b_2(N - 1)$, where b_2 reflects the cost component that increases in the number of grades N offered. As one would anticipate, separation is optimal if b_2 is very high and complete classification is optimal if $b_2 = 0$.

PROPOSITION 4. *There exists a threshold for b_2 above which the separation strategy is optimal in both the direct and indirect channel settings (i.e., $N^{*I} = N^{*D} = 1$). Furthermore, assuming the production output distribution F has an increasing failure rate (IFR), there is a unique optimal production quantity and product quality for the manufacturer in each setting such that*

(*i*) *if the production cost $C_p(Q)$ is linear, $Q^{*I} = \frac{1}{2}Q^{*D}$ and $x_1^{*I} = x_1^{*D}$;*
(*ii*) *if the production cost $C_p(Q)$ is convex, $\frac{1}{2}Q^{*D} < Q^{*I} < Q^{*D}$ and $x_1^{*I} \geq x_1^{*D}$.*

The optimal quality is higher in an indirect channel than in a direct channel setting when the production cost is convex because the quantity-reduction force is less strong than in the linear-cost case. A similar finding holds in the complete-classifications strategy.

PROPOSITION 5. *If the classification cost $b_2 = 0$ then complete classification is optimal for the manufacturer in both the indirect and direct channel settings (i.e., $N^{*I} = N^{*D} = \infty$). Moreover, there is a unique optimal production quantity and product line design in each setting such that*

(*i*) *if the production cost $C_p(Q)$ is linear, $Q^{*I} = \frac{1}{2}Q^{*D}$, $x_1^{*I} = x_1^{*D}$ and $x_N^{*I} = x_N^{*D} = \bar{x}$.*
(*ii*) *if the production cost $C_p(Q)$ is strictly convex, $\frac{1}{2}Q^{*D} < Q^{*I} < Q^{*D}$, $x_1^{*I} \geq x_1^{*D}$ and $x_N^{*I} = x_N^{D} = \bar{x}$.*

In complete classification, the quality of the highest grade is set to the upper support $\bar{x}$ of the output distribution in both channel settings. The quality of the lowest grade can be strictly higher than the lower support. For a linear production cost the quality of the lowest grade is identical in both channels but if the production cost is convex then the quality of the lowest grade is higher in an indirect channel as compared to a direct channel. Recalling that the product line length is defined as the difference in quality between the highest and lowest grades, this implies that an indirect channel reduces the product line length as compared to a direct channel, at least when the production cost is convex. With additional assumptions, we can develop closed form expressions for the optimal production quantities and product line designs in both channels:

EXAMPLE 1 (COMPLETE CLASSIFICATION WITH UNIFORM QUALITY DISTRIBUTION). Consider the case in which the quality distribution $F(x)$ is uniform between $[\underline{x}, \bar{x}]$. Define $\mu = \frac{\underline{x}+\bar{x}}{2}$ and $\sigma = \frac{\bar{x}-\underline{x}}{2\sqrt{3}}$ to denote the mean and standard deviation of $F(x)$. Let $C_p(Q) = cQ^2$ and $C_b(Q, N) = 0$. The zero classification cost implies that $b_2 = 0$ and therefore complete classification is optimal. Applying Corollary A1[11] in the Appendix, the optimal production quantities and product lines are as follows:

$$Q^{*I} = \begin{cases} \frac{\mu}{4(\mu-\sigma/\sqrt{3})+2c} \text{ if } c \geq \frac{2\sigma}{\sqrt{3}}; \\ \frac{1}{2}\sqrt[3]{\frac{\sigma}{4\sqrt{3}c}} \text{ if } c < \frac{2\sigma}{\sqrt{3}}; \end{cases} \text{ and}$$

$$Q^{*D} = \begin{cases} \frac{\mu}{2(\mu-\sigma/\sqrt{3}+c)} \text{ if } c \geq \frac{\sigma}{\sqrt{3}}; \\ \frac{1}{2}\sqrt[3]{\frac{\sigma}{\sqrt{3}c}} \text{ if } c < \frac{\sigma}{\sqrt{3}}; \end{cases}$$

$$x_N^{*I} = \mu + \sqrt{3}\sigma \text{ and } x_N^{*D} = \mu + \sqrt{3}\sigma;$$

$$x_1^{*I} = \begin{cases} \mu - \sqrt{3}\sigma \text{ if } c \geq \frac{2\sigma}{\sqrt{3}}; \\ \mu + \sqrt{3}\sigma - (6\sigma)^{2/3}c^{1/3} \quad \text{if } c < \frac{2\sigma}{\sqrt{3}}; \end{cases} \text{ and}$$

$$x_1^{*D} = \begin{cases} \mu - \sqrt{3}\sigma \text{ if } c \geq \frac{\sigma}{\sqrt{3}}; \\ \mu + \sqrt{3}\sigma - (2)(3\sigma)^{2/3}c^{1/3} \text{ if } c < \frac{\sigma}{\sqrt{3}}. \end{cases}$$

The product line length distortion $x_N^{*D} - x_1^{*D} - (x_N^{*I} - x_1^{*I})$ is then given by the following:

$$x_1^{*I} - x_1^{*D} = \begin{cases} (2^{1/3} - 1)(6\sigma)^{2/3}c^{1/3} \text{ if } c < \frac{\sigma}{\sqrt{3}}; \\ 2\sqrt{3}\sigma - (6\sigma)^{2/3}c^{1/3} \quad \text{if } \frac{\sigma}{\sqrt{3}} \leq c < \frac{2\sigma}{\sqrt{3}}; \\ 0 \text{ if } c \geq \frac{2\sigma}{\sqrt{3}}. \end{cases}$$

This distortion is nonnegative, that is, the indirect channel reduces length. The distortion first increases and then decreases in the convex cost coefficient c. It eventually goes to zero because the manufacturer will not discard any output, that is, $x_1^{*I} = x_1^{*D} = \underline{x}$, if the production cost is prohibitively high.

To this point in our exploration of the effect of an indirect channel, we have assumed that the product line size, that is, the number of grades offered N, was either (a) exogenously fixed and equal in both channels, or (b) the optimal size was invariant to the channel as was the case for the extreme strategies of separation ($N^* = 1$ if b_2 very high) and complete classification ($N^* = \infty$ if $b_2 = 0$). We now explore the effect of the indirect channel on the optimal product line size and optimal product line length for general values of the classification cost b_2.

The manufacturer's problem (1) of optimizing the quantity Q and product line $\boldsymbol{x}$ is a concave maximization problem over $N + 2$ variables, where the product line size N is itself a decision. For any given quantity Q, the optimal size N^* and the optimal specification vector $\boldsymbol{x}^*$ can be determined by solving a shortest path problem as described in the Appendix. The optimal size N^* for a given quantity Q corresponds to the number of edges in the shortest path. This is not amenable to a closed form characterization, and therefore neither is the global optimal N^* (i.e., over all possible Q) except in the earlier extreme cases in which the per-grade classification cost b_2 is either very high (so that $N^* = 1$) or 0 (so that $N^* = \infty$). Nonetheless, we are able to develop below an analytical result as to whether the optimal size is higher or lower in an indirect channel as compared to a direct channel.

At any given fixed production quantity, the manufacturer can increase its revenue by offering more grades (see Lemma A1 in the Appendix) but doing so increases its classification cost if $b_2 > 0$. Because the manufacturer only earns a portion of the end-customer revenue in an indirect channel, it will therefore offer fewer grades in an indirect channel than in a direct channel at any fixed production quantity. As established in the following proposition, this product line size reduction holds even at the channel optimal production quantities and product line designs.

Proposition 6. *If the production cost $C_p(Q)$ is linear then the size of the optimal product line in an indirect channel is lower than in a direct channel, that is,* $N^{*I} \leq N^{*D}$.

We note that Villas-Boas (1998) finds a similar result in the independent product setting when two products are offered to serve two customer segments: the number of products in the product line might be reduced to one in the indirect channel. The reason for the size reduction in Villas-Boas (1998) lies in the fact that the indirect channel induces the manufacturer to reduce the quality of the lower-quality product with the result that it (sometimes) becomes unattractive to offer. That is not the driver of size reduction in our co-product setting because as we showed above the indirect channel does not necessarily induce the manufacturer to reduce the quality of the lowest grade. The size reduction effect is driven (at least in part) by a classification cost that increases in the number of grades offered, a product line size-related cost that was not considered by Villas-Boas (1998). In addition, Aydin and Hausman (2009) derive a similar result for horizontally differentiated products but they focus on the distributor's assortment selection while assuming the manufacturer's product line is exogenously given.

The fact that an indirect channel results in a smaller sized product line will impact the length of the resulting product line. The smaller size will exert downward pressure on the quality of the highest grade; intuitively, one would expect the quality of the highest grade in a two-product line to be higher than the grade quality in a single product line. Also, the production quantity is influenced by the size of the product line, and so the size influences the product line qualities indirectly through the production quantity force discussed above.

We now present the numerical studies used to investigate the effect of the indirect channel on the length and size of the optimal product line when the classification cost component b_2 is positive (i.e., a finite number of grades is optimal) but not so large as to induce the separation strategy (i.e., a single-grade line is not optimal). In the first study, we examine on the *linear production cost* case. We set $C_p(Q) = c_1 Q$ and vary c_1 from 0.02 to 0.14 in increments of 0.02. The output quality spectrum [distribution $F(x)$; density $f(x)$] is set as a truncated normal distribution $N(1, \sigma^2)$ between [0, 2] with σ taking on one of the three values $\{0.2, 0.3, 0.4\}$. For the classification cost $C_b(Q, N)$, we set $b_0 = 0$ and $b_1 = 0.05c_1$ and vary b_2 from 0.0002 to 0.0018 in increments of 0.0004. We solved these 105 instances for both the indirect- and direct-channel settings using the shortest-path algorithm for $\boldsymbol{x}$ together with an exhaustive search for Q, as described in the Appendix. In the second study, we examine the *convex production cost* case. We set $C_p(Q) = c_1 Q + c_2 Q^2$ where $c_1 = 0.06$ and c_2 varied from 0 to 0.25 in increments of 0.05. The other parameters were chosen as in the above linear production cost study and so there are 90 instances in total.

Before reporting the aggregate results for each of the two studies, we first present the optimal product line designs in the direct and indirect channel for one instance of the linear cost structure ($c_1 = 0.06$) and one instance of the convex cost structure ($c_1 = 0.06$, $c_2 = 0.10$). We set $b_2 = 0.0018$ and $\sigma = 0.3$ in both instances. The optimal product lines are shown in Figure 1, where the quality distribution $F(x)$ is overlaid with the optimal grade specifications (illustrated by the vertical lines). Everything to the left of grade 1 is discarded. Grades 1, 2, etc. are given by the bands, with the grade quality specified by the minimum quality in the band. In both the linear and the convex cases, the optimal product line size and length are each lower in the indirect channel than the direct channel. As we now discuss, these directional distortions are not unique to these particular instances.

In all instances of the linear cost study, we observed that the size and the length of the product line were both reduced in the indirect channel as compared to the direct channel. Figure 2a plots the relative reduction in product line length for the indirect channel, that is, $1 - \frac{x_N^{*I} - x_1^{*I}}{x_N^{*D} - x_1^{*D}}$, as a function of b_2 (the cost of offering additional grades). Recall from Proposition 5 that there is no length reduction at $b_2 = 0$ for a linear production cost. However, when $b_2 > 0$, we see that the indirect channel does reduce the product line length even if the production cost is linear; with the average reduction in length being greater than 10% for $b_2 \geq 0.0006$.[12] Figure 2b shows the product line size (averaged across instances) as a function of b_2 for both the indirect and direct channels. Both sizes decrease as the cost of offering additional grades b_2 increases. On average (and in all instances) the size of the product line is lower in the indirect channel than in the direct channel. The absolute size reduction $N^{*D} - N^{*I}$ decreases in b_2 but the relative reduction is approximately 30%, irrespective of the values of b_2. Table 1 summarizes more detailed results from the numerical study.

In all but three instances, we observed an upward distortion in the quality of the lowest grade, indicating that the sales–volume reduction force exerted more upward pressure than the production-quantity reduction force exerted downward pressure.[13] The reason for this is that the manufacturer's optimal quantity in the indirect channel was lower than in the direct channel but greater than 50% of the direct channel quantity (because the product line size differs in the indirect channel). This means that the production-quantity reduction is less severe than the 50% reduction proven above when the product line size was fixed (see Proposition 3), and so the resulting downward pressure on quality is less strong. In all instances, we observed that

Figure 1 Optimal Product Line Designs in Different Channels

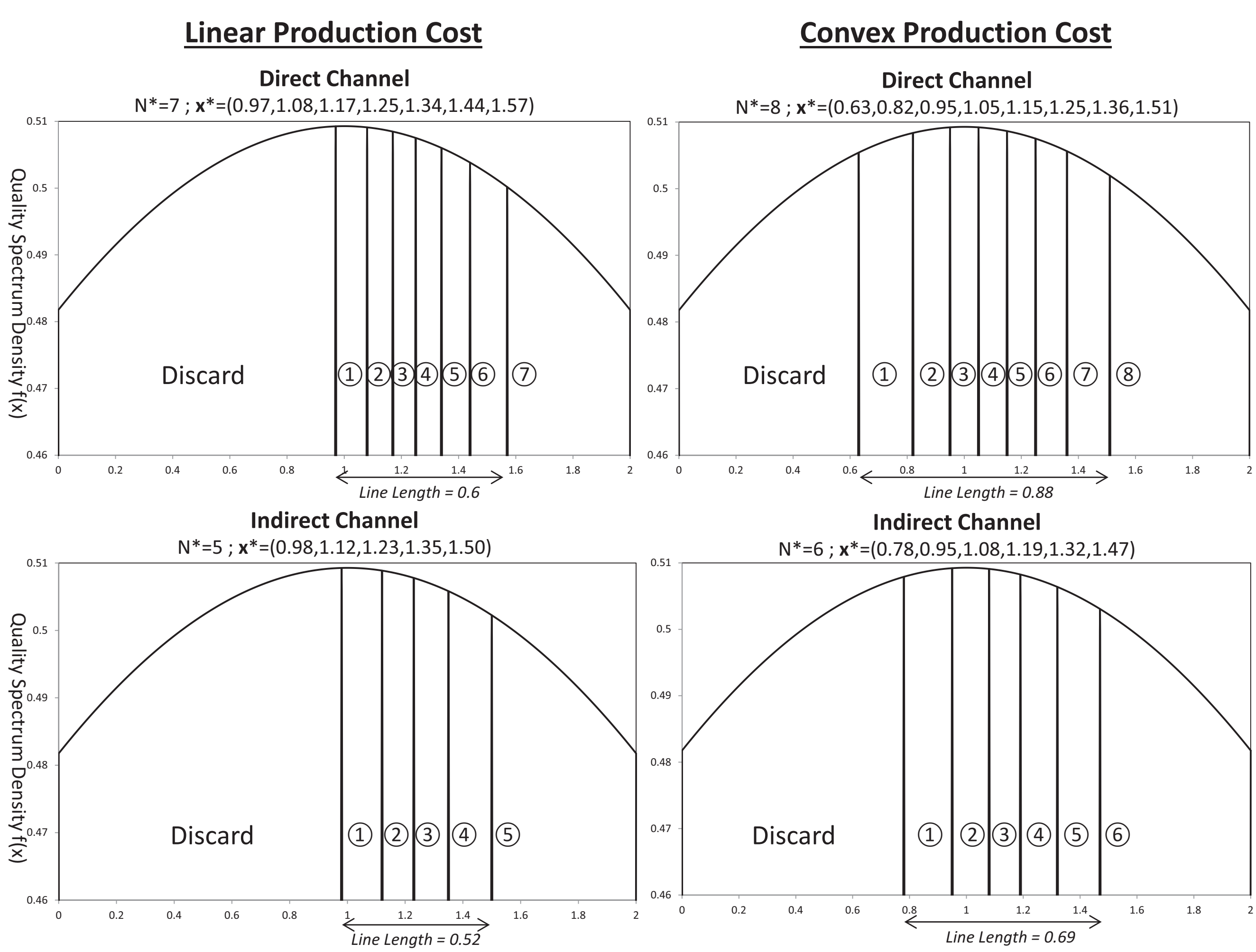
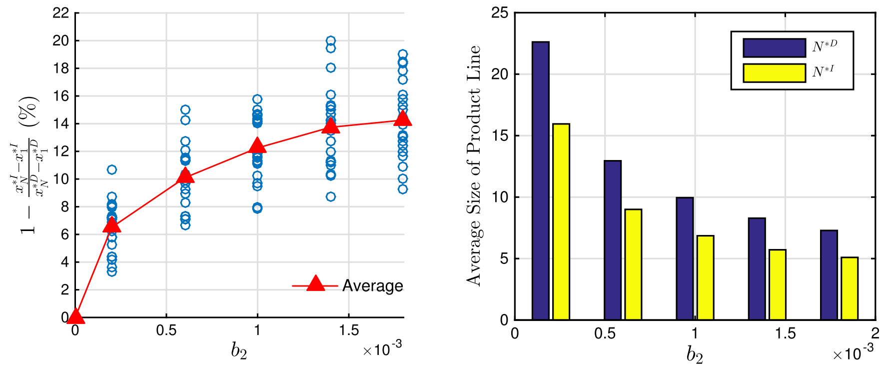


Figure 2 Effect of the Indirect Channel with a Linear Production Cost [Color figure can be viewed at wileyonlinelibrary.com]



(a) Relative decrease in product line length (b) Size of product line

Table 1 Effect of the Indirect Channel with a Linear Production Cost

	b_2	0.0002	0.0006	0.001	0.0014	0.0018
Size	$N^{*I} - N^{*D}$	−6.7	−4.0	−3.1	−2.6	−2.2
	Relative decrease (%)	**29.44**	**30.50**	**31.05**	**30.96**	**29.89**
Length	$x_N^{*I} - x_N^{*D}$	−0.046	−0.056	−0.065	−0.068	−0.068
	$x_1^{*I} - x_1^{*D}$	0.008	0.019	0.020	0.022	0.021
	Absolute decrease	0.054	0.075	0.085	0.090	0.089
	Relative decrease (%)	**6.51**	**10.09**	**12.25**	**13.74**	**14.26**

Note: The numbers are average values among all the instances.

the quality of the highest grade was lower in the indirect channel than in the direct channel. This is a result of the reduced product line size and the associated downward pressure on the highest quality as discussed above. This downward distortion amplifies the reduction in the product line length.

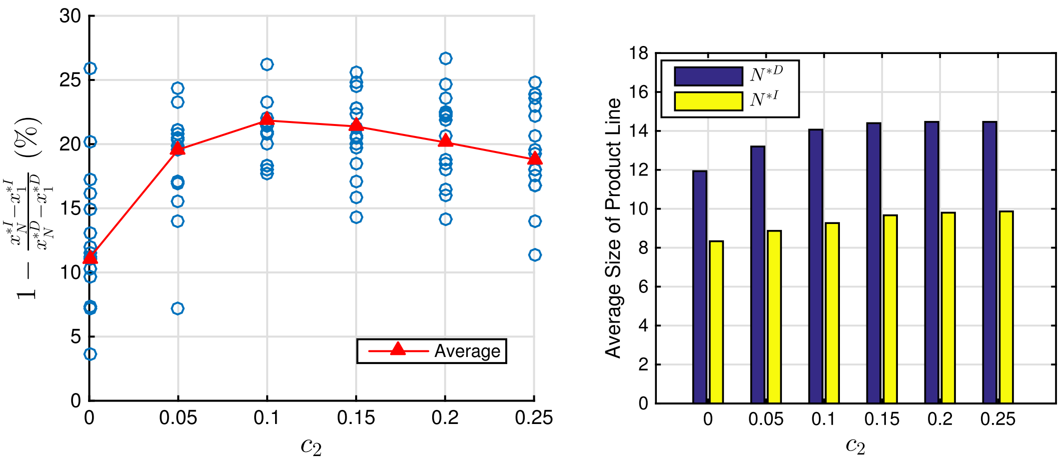
The results of the convex cost study are reported in Figure 3 and Table 2. As shown in Figure 3a, the relative reduction in the product line length (indirect vs. direct channel) initially increases in the convexity parameter c_2 because convexity reduces the production quantity distortion (see Proposition 1), and therefore the sales–volume reduction force (Proposition 2) is stronger than the production-quantity reduction force. Recall that the two forces are equal when the production cost is linear. However, as c_2 continues to increase, the distortion in the product line length gradually reduces because the overall quantity produced (in either channel setting) becomes smaller (and eventually zero) as c_2 becomes larger. This pattern exactly echoes the earlier analytical result for Example 1. Similar to the linear production cost case, the indirect channel reduces the size of the product line by around 30%, but the magnitude of reduction is not very sensitive to c_2. See Figure 3b and Table 2. We note that under a convex production cost, a greater upward distortion occurs in x_1 as compared to the linear case but the downward distortion in x_N is not significantly influenced by the convexity. So, the product line length is reduced more in the convex production cost case.

We also explored a more limited convex-cost setting in which we fixed the product line size in both channels to $N = 2$. We observed that the quality of both grades x_1 and x_2 was higher in the indirect channel, indicating that the sales–volume reduction force exerted more upward pressure on both qualities than the production-quantity reduction exerted downward pressure. However, the product line length, that is, $x_2 - x_1$, was lower in the indirect channel because the upward and downward pressures differentially affect the higher and lower qualities, with a greater net increase in the lower quality than in the higher quality. This shows that the length reduction effect of the indirect channel is not driven only by a size reduction. It can hold even when size is fixed.

4.3. Impact of the Indirect Channel on Profits

We now consider the indirect channel effect on profits. Let Π_m^{*I} and Π_d^{*I} denote the manufacturer's and the distributor's profits in equilibrium in the indirect channel setting, and let Π_m^{*D} be the manufacturer's optimal profit in the direct-channel setting. For the special case of Example 1 (Uniform Quality Distribution, Quadratic Production Cost $C_p(Q) = cQ^2$, and Zero Classification Cost), these profits can be derived

Figure 3 Effect of the Indirect Channel with a Convex Production Cost [Color figure can be viewed at wileyonlinelibrary.com]



(a) Relative decrease in product line length

(b) Size of product line

Table 2 Effect of the Indirect Channel with a Convex Production Cost

	c_2	0	0.05	0.1	0.15	0,2	0,25
Size	$N^{*I} - N^{*D}$	−3.6	−4.3	−4.8	−4.7	−4.7	−4.6
	Relative decrease (%)	**30.50**	**32.91**	**32.32**	**28.96**	**28.96**	**28.96**
Length	$x_N^{*I} - x_N^{*D}$	−0.060	−0.053	−0.054	−0.051	−0.052	−0.052
	$x_1^{*I} - x_1^{*D}$	0.011	0.111	0.160	0.178	0.172	0.159
	Absolute decrease	0.071	0.164	0.214	0.229	0.224	0.211
	Relative decrease (%)	**11.11**	**19.57**	**21.84**	**21.38**	**20.14**	**18.81**

Note: The numbers are average values among all the instances.

in closed form (see Proposition A5 in the Appendix), and are given by the following:

$$\Pi_m^{*I} = \begin{cases} \frac{\mu^2}{4(c+2\mu-2\sigma/\sqrt{3})} & \text{if } c \geq \frac{2\sigma}{\sqrt{3}}, \\ \frac{1}{8}\left(\mu+\sqrt{3}\sigma-\frac{1}{2}(6\sigma)^{2/3}c^{1/3}\right) & \text{if } c < \frac{2\sigma}{\sqrt{3}}, \end{cases}$$

$$\Pi_d^{*I} = \begin{cases} \frac{\mu^2(\mu-\sigma/\sqrt{3})}{4(c+2\mu-2\sigma/\sqrt{3})^2} & \text{if } c \geq \frac{2\sigma}{\sqrt{3}}, \\ \frac{1}{16}\left(\mu+\sqrt{3}\sigma-\frac{2}{3}(6\sigma)^{2/3}c^{1/3}\right) & \text{if } c < \frac{2\sigma}{\sqrt{3}} \end{cases}$$

$$\text{and } \Pi_m^{*D} = \begin{cases} \frac{\mu^2}{4(c+\mu-\sigma/\sqrt{3})} & \text{if } c \geq \frac{\sigma}{\sqrt{3}}, \\ \frac{1}{4}\left(\mu+\sqrt{3}\sigma-(3\sigma)^{2/3}c^{1/3}\right) & \text{if } c < \frac{\sigma}{\sqrt{3}}, \end{cases}$$

In the indirect channel setting, the ratio of the manufacturer's profit to the retailer's can be written as

$$\frac{\Pi_m^{*I}}{\Pi_d^{*I}} = \begin{cases} \frac{c+2\mu-2\sigma/\sqrt{3}}{\mu-\sigma/\sqrt{3}} & \text{if } c \geq \frac{2\sigma}{\sqrt{3}}, \\ 2+\frac{\frac{1}{3}(6\sigma)^{2/3}c^{1/3}}{\mu+\sqrt{3}\sigma-\frac{2}{3}(6\sigma)^{2/3}c^{1/3}} & \text{if } c < \frac{2\sigma}{\sqrt{3}}, \end{cases} \tag{8}$$

which is increasing in the production cost convexity parameter c. It then follows that the manufacturer earns a larger portion of the total supply chain profit as the production cost becomes more convex. The efficiency of the indirect channel (i.e., its profit relative to the direct channel, or equivalently, relative to a centrally-optimized indirect channel) is given by the profit ratio:

$$\frac{\Pi_m^{*I}+\Pi_d^{*I}}{\Pi_m^{*D}} = \begin{cases} \frac{3}{4}+\frac{(9-(5)(2)^{2/3})(3\sigma)^{2/3}c^{1/3}}{12[\mu+\sqrt{3}\sigma-(3\sigma)^{2/3}c^{1/3}]} & \text{if } c < \frac{\sigma}{\sqrt{3}}; \\ \frac{(c+\mu-\sigma/\sqrt{3})(9\mu+9\sqrt{3}\sigma-5(6\sigma)^{2/3}c^{1/3})}{12\mu^2} & \text{if } \frac{\sigma}{\sqrt{3}} \leq c < \frac{2\sigma}{\sqrt{3}}; \\ 1-\frac{3(\sqrt{3}\mu+\sigma)^2}{(3c+6\mu-2\sqrt{3}\sigma)^2} & \text{if } c \geq \frac{2\sigma}{\sqrt{3}} \end{cases} \tag{9}$$

The indirect channel efficiency increases in the production cost convexity parameter c for $c < \frac{\sigma}{\sqrt{3}}$, as does the manufacturer's portion of the total supply chain profit. Interestingly, we established earlier that the product line length distortion increased in c for $c < \frac{\sigma}{\sqrt{3}}$. Putting these findings together, it follows that an increase in the product line length distortion does not necessarily hurt the manufacturer or the overall supply chain. As we will now show numerically, this finding that a larger distortion in length is not necessarily associated with a larger loss in efficiency holds beyond the specific assumptions of Example 1.

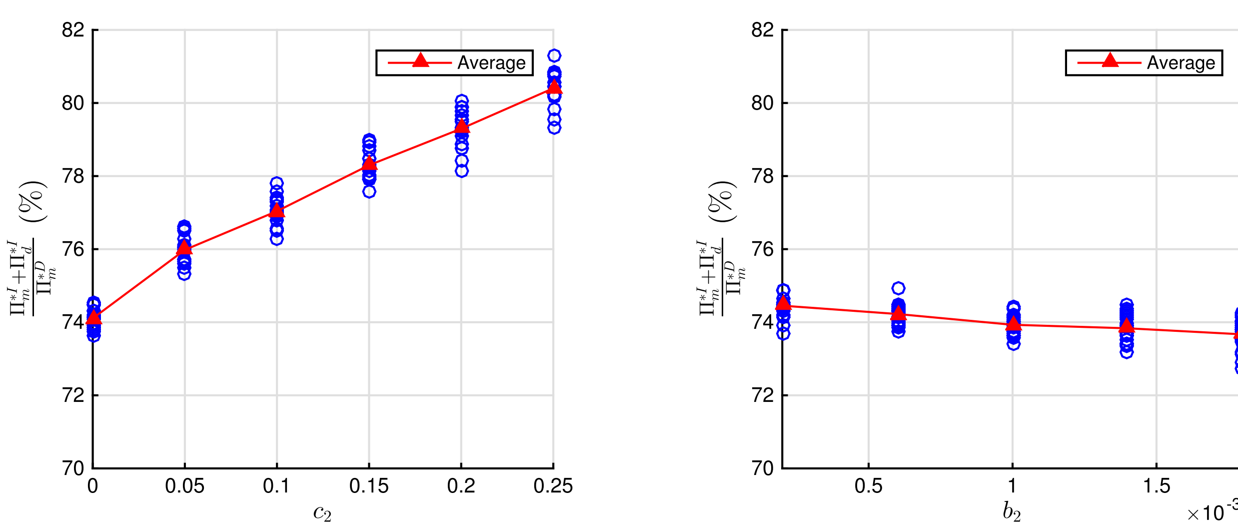
We calculated the indirect channel efficiency for each of the instances of the linear-cost and convex-cost studies described above in section 4.2. For the convex-cost study, Figure 4a plots the efficiency as a function of the production cost convexity parameter c_2. Observe that the efficiency increases in c_2. We observed earlier that the length distortion initially increased in c_2 (see Figure 3a), indicating that a larger length distortion does not necessarily imply a lower efficiency. The observation that a greater product line distortion does not necessarily imply a lower efficiency can be explained by the fact that efficiency is also influenced by the production quantity distortion. Convexity in the production cost may increase the distortion in the product line but it reduces the quantity distortion. Our results suggest that quantity distortion has more of an effect on the indirect channel efficiency. For the linear-cost study, Figure 4b plots the efficiency as a function of the grade-driven classification cost b_2. The efficiency decreases in b_2. We observed earlier that the length distortion increased in b_2 (see Figure 2a), and so here we see that a larger length distortion is in fact associated with a lower efficiency for linear production cost.

In addition, Figure 5a and b depict the manufacturer's portion of the overall indirect channel profit for the convex and linear cost studies. The effects of b_2 and c_2 are similar to their effects on supply chain efficiency seen in Figure 4a and b.

Finally, we consider a manufacturer who naively adopts the optimal direct-channel product line design even when it sells exclusively through a distributor. (We assume the manufacturer uses the optimal indirect channel production quantity given this product line design). Let $\hat{\Pi}_m^{*I}$ denote the manufacturer's profit with this suboptimal strategy. Recalling that Π_m^{*I} and Π_m^{*D} denote the manufacturer's optimal profits in the indirect and direct channel cases respectively, then the following two ratios $\frac{\Pi_m^{*I}-\hat{\Pi}_m^{*I}}{\hat{\Pi}_m^{*I}}$ and $\frac{\Pi_m^{*I}-\hat{\Pi}_m^{*I}}{\Pi_m^{*D}}$ reflect two different values of product line flexibility for the manufacturer. The first ratio measures the value in the indirect channel of being able to deploy the optimal product line. The second ratio measures the percentage of the direct channel profit that can be recovered through product line redesign by a manufacturer who moves from a direct to an indirect channel strategy.

We computed the values of $\hat{\Pi}_m^{*I}$ for all instances of the numerical studies in section 4.2, and the average values of $\frac{\Pi_m^{*I}-\hat{\Pi}_m^{*I}}{\hat{\Pi}_m^{*I}}$ and $\frac{\Pi_m^{*I}-\hat{\Pi}_m^{*I}}{\Pi_m^{*D}}$ are summarized in Table 3.[14] The first ratio indicates that when the production cost is strictly convex or the classification cost b_2 is sufficiently high, the manufacturer can improve its profit by more than 1% by adopting the optimal indirect channel product line design rather than naively using the direct-channel design. The second ratio is generally smaller than the first one because the indirect channel causes a substantial profit loss as compared to a direct channel. The effects of the classification cost and production convexity parameters on these ratios generally mirror their effects on product line length distortion as reported in Tables 1 and 2: the larger is the distortion then the greater the value of flexibility. It is worth noting that because we

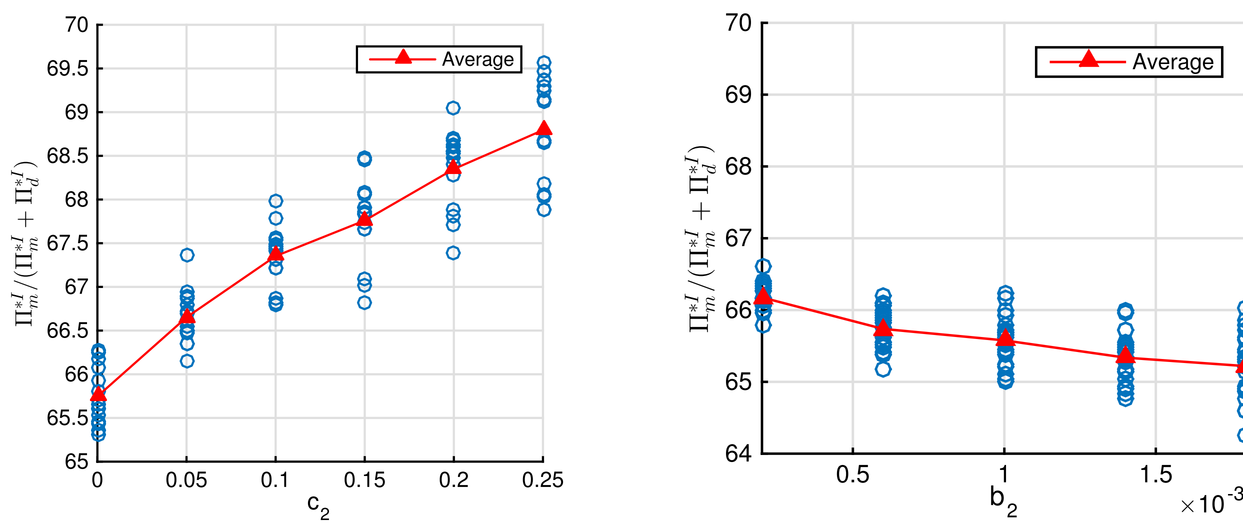
Figure 4 Indirect Channel Efficiency [Color figure can be viewed at wileyonlinelibrary.com]



(a) Impact of the convex cost coefficient c_2 (convex production cost)

(b) Impact of the classification cost b_2 (linear production cost)

Figure 5 Impact of the Indirect Channel on the Manufacturer's Profit Portion [Color figure can be viewed at wileyonlinelibrary.com]



(a) Impact of the convex cost coefficient c_2 (convex production cost)

(b) Impact of the classification cost b_2 (linear production cost)

normalize fixed costs to zero in our model, our "profit" is technically "contribution" in accounting terminology. If fixed costs were included then the percentages reported in this table would in fact be larger.

Table 3 The Value of Flexibility in Product Line Design

			Linear production cost			
b_2	0.0002	0.0006	0.001	0.0014	0.0018	
$\frac{\Pi_m^{*I}-\hat{\Pi}_m^{*I}}{\hat{\Pi}_m^{*I}}$ (%)	0.33	0.61	0.81	0.92	1.10	
$\frac{\Pi_m^{*I}-\hat{\Pi}_m^{*I}}{\Pi_m^{*D}}$ (%)	0.16	0.30	0.39	0.44	0.52	
			Convex production cost			
c_2	0	0.05	0.1	0.15	0.2	0.25
$\frac{\Pi_m^{*I}-\hat{\Pi}_m^{*I}}{\hat{\Pi}_m^{*I}}$ (%)	0.69	1.15	1.35	1.42	1.34	1.26
$\frac{\Pi_m^{*I}-\hat{\Pi}_m^{*I}}{\Pi_m^{*D}}$ (%)	0.33	0.57	0.69	0.74	0.71	0.69

Note: The results are average values among all the instances.

5. Extensions

In what follows, we consider three different extensions to our base model. We first examine the robustness of our findings when the customer population is not uniformly distributed. We then consider alternative production and classification cost structures. Finally, we examine whether there exists a theoretical contract that coordinates the channel in the base model.

5.1. Nonuniform Customer Types

In our base model we assumed that the customer valuation θ was uniformly distributed. We now relax this assumption by considering a more general distribution $G(\theta)$ on a support $[\underline{\theta}, \bar{\theta}]$. An important foundation of our earlier analysis relied on the transformation of the bilevel program into a single-stage optimization problem by using the property that the distributor's problem can be equivalently represented by its KKT conditions; see section 4.1. In fact, as we establish in section S.3 of Appendix S1, the uniform distribution assumption is not required for this distributor-problem property. The property holds if $G(\theta)$ has an increasing failure rate (IFR). Furthermore, the manufacturer's problem is well-behaved if the following regularity condition is imposed on $G(\theta)$: $\frac{3\bar{G}(\theta)}{g(\theta)} - \theta + (\frac{\bar{G}(\theta)}{g(\theta)})^2 \frac{g'(\theta)}{g(\theta)}$ is nonincreasing in θ. This condition is satisfied by a number of nonuniform distributions (see Lemma S3 in Appendix S.3), including the power distribution $G(\theta) = 1 - (1 - \theta)^{\gamma}$ where $\theta \in [0, 1]$ and $\gamma > 0$. When $\gamma = 1$, the power distribution reduces to the uniform distribution studied in the base model. When $\gamma > 1$, more customers reside at lower-end valuations. When $0 \le \gamma < 1$, more customers reside at higher-end valuations. We note that power distribution satisfies the IFR property.

When the per-grade classification cost $b_2 = 0$, and so complete classification is optimal, the following proposition proves that our earlier result (Proposition 5) that the indirect channel has a shorter product line length than the direct channel still holds when θ follows a power distribution.

Proposition 7. *Let* $G(\theta) = 1 - (1 - \theta)^{\gamma}$ *where* $\theta \in [0, 1]$ *and* $\gamma > 0$. *Define* $\zeta = (\frac{1+\gamma}{\gamma})^{\gamma}$. *If* $b_2 = 0$ *then* *complete classification is optimal for both channel settings, that is,* $N^{*I} = N^{*D} = \infty$*, and furthermore*

(*i*) *If* $C_p(Q)$ *is linear,* $Q^{*I} = \frac{1}{\zeta}Q^D$. $x_N^{*I} = x_N^{*D} = \bar{x}$, $x_1^{*I} = x_1^{*D}$ *and* $x_N^{*I} = x_N^{*D} = \bar{x}$;

(*ii*) *if* $C_p(Q)$ *is strictly convex,* $\frac{1}{\zeta}Q^{*D} < Q^{*I} < Q^{*D}$, $x_1^{*I} \geq x_1^{*D}$ *and* $x_N^{*I} = x_N^{*D} = \bar{x}$.

When $b_2 > 0$, our earlier result that the optimal line size is lower in the indirect channel (Proposition 6) continues to hold for the power distribution; see Proposition S1 in Appendix S.3 which also establishes that Propositions 1 and 3 continue to hold.

We conducted the following numerical study to examine the channel effect on line length and size when $b_2 > 0$. We set $G(\theta) = 1 - (1 - \theta)^\gamma$ where γ varies from 0.4 to 1.6 in increments of 0.3. The quality distribution F is set as a truncated normal distribution $N(1, \sigma^2)$ between [0, 2] with σ taking on one of the three values {0.2, 0.3, 0.4}. We set $c_1 = 0.06$, $c_2 \in \{0, 0.1, 0.2\}$ and $b_2 \in \{0.0006, 0.001, 0.0014\}$. The results are summarized in Figure 6 and Table 4. The earlier key findings that product line length and size are lower in the indirect channel still hold when θ is not uniformly distributed ($\gamma \neq 1$). Interestingly, we observe that the product line distortion is smaller when more customers have higher quality valuations (i.e., γ is smaller). Intuitively, this observation might be explained by the following. Grade specifications tend to be closer to each other as more customers are concentrated at high valuations, and this then limits the magnitude of the product line distortion caused by the indirect channel.

5.2. Alternative Cost Structures

We now return to the uniform population distribution assumption but consider alternative production and classification cost structures.

5.2.1. Concave Production Cost. We have assumed to this point that the production cost function $C_p(Q)$ was weakly convex in the production quantity Q. There may be settings, however, in which production exhibits economies of scale such that $C_p(Q)$ is concave. Although, the manufacturer's profit function is not well-behaved (i.e., unimodal) in general for concave cost functions, we can analytically characterize the optimal solution with certain conditions imposed:

PROPOSITION 8. *Assume that* $b_2 = 0$ (*i.e., complete classification is optimal*), $C_p(Q) = c_1Q + c_2Q^\beta$ *where* $c_1, c_2 \geq 0$ *and* $0 < \beta < 1$ (*i.e., production cost is concave*), *and the quality distribution* $F(x)$ *is uniform with mean* μ *and standard deviation* σ.

If $b_1 + c_1 + 4^{1-\beta}\beta c_2 \leq \sigma/\sqrt{3}$, *then* $\frac{1}{4} < Q^{*I} < \frac{1}{2}Q^{*D}$ *and* Q^{*I} *and* Q^{*D} *are uniquely determined by the following equations, respectively:*

$$b_1 + c_1 = \frac{\sigma}{16\sqrt{3}(Q^{*I})^2} - \frac{c_2\beta}{(Q^{*I})^{1-\beta}};$$

$$b_1 + c_1 = \frac{\sigma}{4\sqrt{3}(Q^{*D})^2} - \frac{c_2\beta}{(Q^{*D})^{1-\beta}}.$$

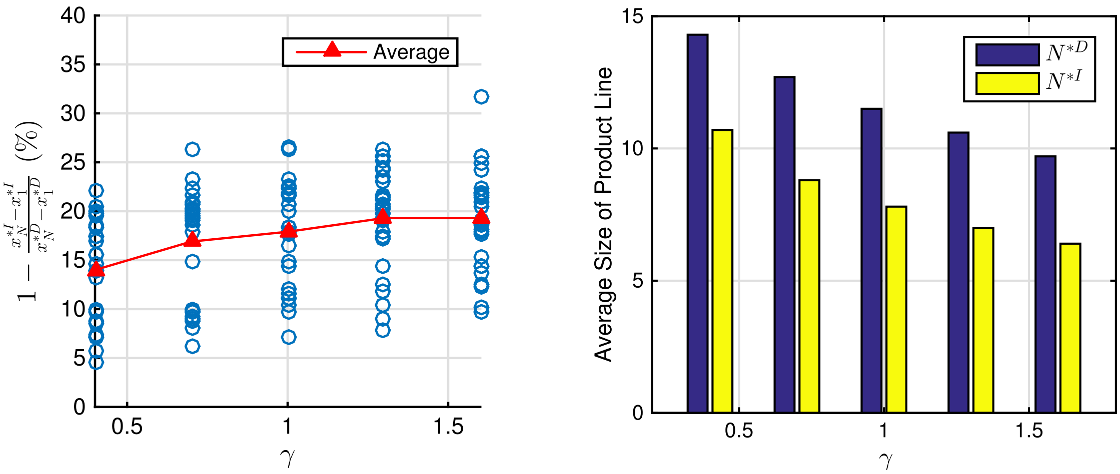
Moreover, $x_N^{*I} = x_N^{*D} = \mu + \sqrt{3}\sigma$, $x_1^{*I} = \mu + \sqrt{3}\sigma(1 - \frac{1}{2Q^{*I}})$ *and* $x_1^{*D} = \mu + \sqrt{3}\sigma(1 - \frac{1}{Q^{*D}})$. *Consequently, the product line length in the indirect channel is greater than that in the direct channel, that is,* $x_N^{*I} - x_1^{*I} > x_N^{*D} - x_1^{*D}$.

That is, when $b_2 = 0$, that is, complete classification is optimal, a concave production cost function can reverse our earlier finding (Proposition 5) that an indirect channel reduces the product line length when the production cost is convex. The reason is that concavity makes the production quantity distortion more severe (i.e., greater than 50%) and therefore the quantity reduction force is stronger than the sales–volume reduction force such that the lowest quality is adjusted downward in compensation for the production quantity reduction. We emphasize that this possible reversal was proven for the special case of $b_2 = 0$.

In the more general case of $b_2 > 0$, the indirect channel will have a lower optimal product line size (i.e., number of grades) than the direct channel. Different from the severe quantity distortion force induced by the concave cost function, this size reduction acts as a force to reduce the product line length. To examine the net effect on the product line, we conducted a numerical study as follows. We set the production cost as $C_p(Q) = c_1Q + c_2Q^\beta$, with $c_1 = 0.06$, $c_2 = 0.1$, and β varied from 0.1 to 0.9 in increments of 0.2. The quality distribution was assumed to be truncated Normal $N(1, \sigma^2)$ on [0, 2] with $\sigma \in \{0.1, 0.2, 0.3\}$. We set $b_0 = 0$, $b_1 = 0.05c_1$ and varied b_2 from 0.0002 to 0.0018 in increments of 0.0002. In most of these 135 instances, the optimal product line length was found to be shorter in the indirect channel than in the direct channel; see Figure 7a and Table 5. Furthermore, the length reduction increases as b_2 becomes larger. This implies that the reduction in the line size (Figure 7b) has a greater impact on the product line length than does the concave production cost, especially when the grade-related classification is high.

In summary, our finding that the indirect channel has a lower product line length than the direct channel is quite robust even when the production cost is concave. However, this channel effect can be reversed at times if the grade-related classification cost b_2 is sufficiently low.

Figure 6 Effect of the Indirect Channel for Customer Distribution $G(\theta) = 1 - (1 - \theta)^{\gamma}$ [Color figure can be viewed at wileyonlinelibrary.com]



(a) Relative decrease in product line length **(b)** Size of product line

Table 4 Effect of the Indirect Channel for Customer Distribution $G(\theta) = 1 - (1 - \theta)^{\gamma}$

	γ	0.4	0.7	1	1.3	1.6
Size	$N^{*D} - N^{*I}$	−3.6	−3.9	−3.7	−3.6	−3.4
	Relative decrease (%)	**25.07**	**30.35**	**32.11**	**34.33**	**34.45**
Length	$x_N^{*I} - x_N^{*D}$	−0.036	−0.051	−0.056	−0.065	−0.068
	$x_1^{*I} - x_1^{*D}$	0.092	0.107	0.113	0.114	0.113
	Absolute decrease	0.128	0.158	0.169	0.179	0.181
	Relative decrease (%)	**14.05**	**16.93**	**17.90**	**19.26**	**19.27**

Note: The numbers are average values among all the instances.

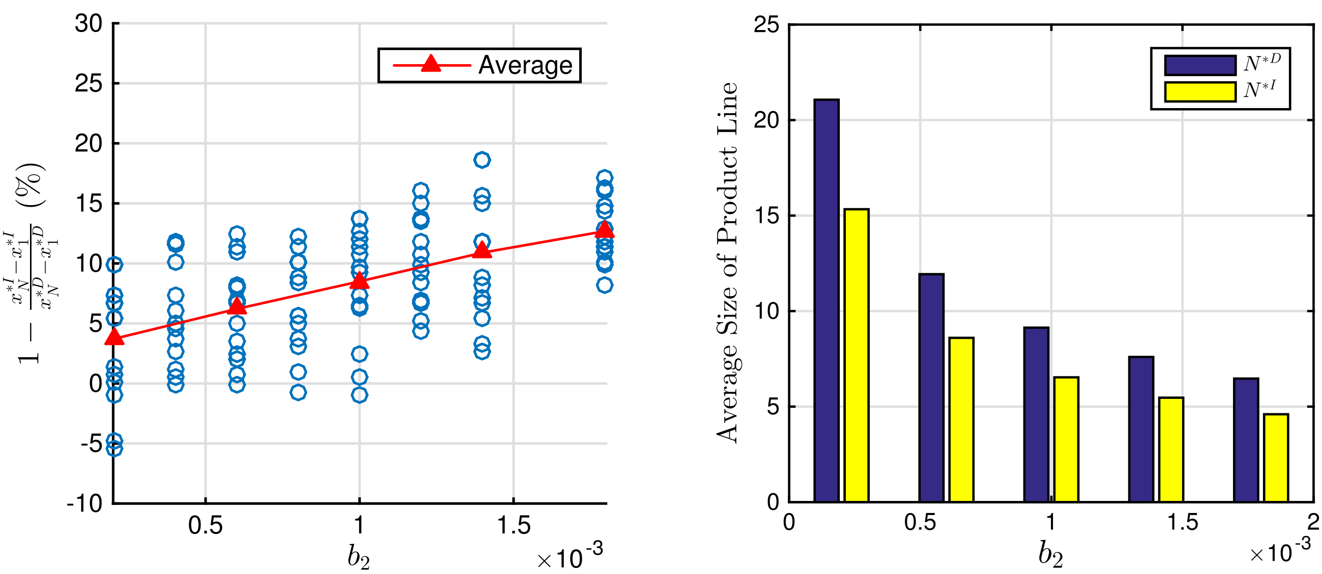
5.2.2. Alternative Classification Cost Structures. In our base model, the classification cost has the form $C_b(Q, N) = b_0 + b_1Q + b_2(N - 1)$; that is, it is linear and separable in the both the quantity Q to classify and the number of grade N. Let us first consider the linearity assumptions before addressing separability.

Suppose that $C_b(Q, N) = b_0 + B_1(Q) + b_2(N - 1)$ where $B_1(Q)$ is an increasing nonlinear function of Q. If the sum of $B_1(Q)$ and the production cost $C_p(Q)$ remains convex then our analysis for the base model readily extends and the main results are unchanged. Alternatively, if $B_1(Q) + C_p(Q)$ is concave in Q, then the analysis and findings in section 5.2.1 apply. Next suppose that $C_b(Q, N) = b_0 + b_1Q + B_2(N)$, where $B_2(N)$ is an increasing nonlinear function of N. We first note that the separation and complete classification Propositions 4 and 5 remain intact because they speak to the special cases when $B_2(N)$ is prohibitively large even for $N = 2$ (Proposition 4) or $B_2(N) = 0$ for any N (Proposition 5). We can also generalize our earlier size-reduction result (Proposition 6):

Proposition 9. *If $C_b(Q, N) + C_p(Q)$ can be written as $B_2(N) + cQ$ where $B_2(N)$ is a strictly increasing function of N and c is a positive scalar, then the size of the optimal product line in an indirect channel is lower than in a direct channel, that is, $N^{*I} \le N^{*D}$.*

The separable cost structure in our base model is predicated on the idea that there is a testing cost for evaluating the quality of each unit (so its grade can be determined) and that there is a separate logistics and marketing cost that scales in the number of grades offered. While we think this is a reasonable approximation to reality, the classification cost may not be separable in Q and N in all settings. To examine the robustness of our findings, we consider the classification cost $C_b(Q, N) = b_1Q + b_2(N - 1) + b_3Q(N - 1)$ where $b_3 > 0$. While we are not able to fully extend our analysis, all the earlier results established for fixed Q or N remain intact. Therefore, the product line design problem can still be reduced to a shortest-path problem for any given Q, and the optimal production quantity can be found by an exhaustive search. We tested two sets of instances for linear and convex production costs, respectively. For the case of a linear production cost, we set $c_1 \in \{0.06, 0.08, 0.1\}$, $b_2 \in \{0.0002, 0.0006, 0.001,$

Figure 7 Effect of the Indirect Channel with a Concave Production Cost [Color figure can be viewed at wileyonlinelibrary.com]



(a) Relative decrease in product line length **(b)** Size of product line

Table 5 Effect of the in Effect of the Indirect Channel with a Concave Production Cost with a Concave Production Cost

	b_2	0.0002	0.0006	0.001	0.0014	0.0018
Size	$N^{*D} - N^{*I}$	5.7	3.3	2.6	2.1	1.9
	Relative decrease (%)	27.10	27.31	28.50	28.53	27.82
Length	$x_N^{*I} - x_N^{*D}$	−0.045	−0.054	−0.057	−0.060	−0.061
	$x_1^{*I} - x_1^{*D}$	−0.019	−0.011	−0.009	−0.003	0.004
	Absolute decrease	0.027	0.042	0.049	0.057	0.065
	Relative decrease (%)	3.75	6.20	8.50	10.93	12.68
	Percentage of instances with length decrease (%)	**80.0**	**93.3**	**93.3**	**100**	**100**

0.0014, 0.0018}, $b_3 \in \{0.0005, 0.001\}$, $\sigma \in \{0.2, 0.3, 0.4\}$. For the case of a convex production cost, we set $c_1 = 0.06$, $c_2 \in \{0.05, 0.1, 0.15, 0.2, 0.25\}$, $b_2 \in \{0.0002, 0.001, 0.0018\}$, $b_3 \in \{0.0005, 0.001\}$, $\sigma \in \{0.2, 0.3, 0.4\}$. Other parameters are the same as in the numerical study of the base model. As reported in Tables 6 and 7, our main results that length and size are lower in the indirect channel are robust when the classification cost is not separable in Q and N.

5.3. Alternative Contracts

We assumed that the manufacturer offered a wholesale price contract in our base model. This was motivated both by the observation in the literature that wholesale price contracts (also known as price-only contracts) "are common in supply chain management practice" (Kayiş et al. 2013, p. 46) and by our correspondence with one large co-product manufacturer who indicated that basic wholesale price contracts are used at times with distributors. Our earlier analysis established that, under a wholesale price contract, the indirect channel creates distortions in the optimal production quantity and the product line design as compared to a centrally-optimized (or, equivalently, direct) channel. These distortions reduce the total channel profit. In this subsection, we explore whether alternative contract forms might coordinate the channel and thus resolve the aforementioned distortions and profit loss.

To coordinate assortment decisions in independent product supply chains, Aydin and Hausman (2009) examine the use of slotting fees in which the manufacturer pays the distributor a fixed fee per product carried to incentivize the distributor to carry a larger product line. In our co-product setting, however, the manufacturer incurs the entire classification cost but receives only a portion of the revenue; and this causes the manufacturer to reduce the product line size as compared to a direct channel (Proposition 6). This suggests that reverse slotting fees in which the distributor pays the manufacturer a fixed fee per grade offered might alleviate, and possibly eliminate, the size distortion. However, even if the size distortion is eliminated, the quantity and length distortions may remain, as we

Table 6 Effect of the Indirect Channel with a Nonseparable Classification Cost and a Linear Production Cost

	b_2	0.0002	0.0006	0.001	0.0014	0.0018
Size	$N^{*D} - N^{*I}$	−1.6	−1.9	−1.9	−1.8	−1.7
	Relative decrease (%)	**12.78**	**19.94**	**23.32**	**24.54**	**26.57**
Length	$x_N^{*I} - x_N^{*D}$	−0.022	−0.035	−0.046	−0.049	−0.060
	$x_1^{*I} - x_1^{*D}$	0.005	0.016	0.019	0.020	0.028
	Absolute decrease	0.027	0.051	0.065	0.069	0.089
	Relative decrease (%)	**3.89**	**7.63**	**10.20**	**11.57**	**13.91**

Note: The numbers are average values among all the instances.

Table 7 Effect of the Indirect Channel with a Nonseparable Classification Cost and a Convex Production Cost

	c_2	0	0.05	0.1	0.15	0.2	0.25
Size	$N^{*D} - N^{*I}$	−1.6	−2.6	−2.8	−3.1	−3.1	−2.9
	Relative decrease (%)	**20.00**	**26.76**	**26.79**	**28.49**	**27.55**	**26.00**
Length	$x_N^{*I} - x_N^{*D}$	−0.039	−0.038	−0.039	−0.042	−0.043	−0.041
	$x_1^{*I} - x_1^{*D}$	0.014	0.106	0.143	0.162	0.152	0.136
	Absolute decrease	0.053	0.144	0.182	0.204	0.195	0.177
	Relative decrease (%)	**9.05**	**18.09**	**20.31**	**20.80**	**19.10**	**17.11**

Note: The numbers are average values among all the instances.

observed in the numerical study at the end of section 4.2 when the number of grades was fixed at $N = 2$ in both channels. It is known in the single-product, supply chain contracting literature (e.g., Cachon and Lariviere 2005) that revenue sharing contracts, whereby the distributor pays the manufacturer a wholesale price and a percentage of its revenue, can help mitigate indirect channel quantity distortions. Based on the above observations, we consider a mixed contract that combines revenue sharing and reverse slotting fees.

In particular, we consider a contract form (s, α, w) in which s is the reverse slotting fee paid by the distributor to the manufacturer for each grade offered, $\alpha \in [0, 1]$ is the percentage of the revenue that the distributor shares with the manufacturer, and w is a grade-independent wholesale price that the distributor pays to the manufacturer per unit (of any grade) purchased. The following proposition proves that in the case of a linear production cost, the supply chain can be coordinated with the above contract when the reverse slotting fee s and wholesale price w are appropriately chosen.

Proposition 10. *Assuming $C_p(Q) = cQ$, a contract (s, α, w) coordinates the supply chain when the reverse slotting fee $s = (1 - \alpha)b_2$ and the wholesale price $w = \frac{(1-\alpha)(c+b_1)}{\bar{F}(x_1)}$, where $\alpha \in [0, 1]$.*

In the proposed contract, the value of α can be used to arbitrarily allocate the total profit between the manufacturer and the distributor. If the manufacturer had full bargaining power, then α would be chosen as the unique value that ensures the distributor's profit equaled its reservation value. If the production cost $C_p(Q)$ is convex, then a similar contract structure can be used to coordinate the supply chain, except that the per-unit grade-independent wholesale price w is replaced by a nonlinear total procurement payment $T(q_1^E) = (1 - \alpha)C_p(\frac{q_1^E}{\bar{F}(x_1)})$ that is a function of the quantity of all grades purchased $q_1^E = q_1 + \ldots + q_N$.

We note that an assumption underlying Proposition 10 is that the manufacturer's production and classification costs and the distributor's revenues are all verifiable. If this assumption is not satisfied, then it would be more challenging to coordinate the supply chain. We leave this for future research.

6. Conclusions

In this study, we examined a manufacturer-distributor model in which a co-product manufacturer sells vertically differentiated co-products through a self-interested distributor to quality-sensitive end customers. The manufacturer determines its production quantity, product line design, and wholesale prices. The distributor determines its purchase quantities and retail prices. We analyze this game as a bi-level optimization problem in which the retailer's optimal purchase quantities become a constraint in the manufacturer's optimization problem.

To summarize our key findings, when selling co-products through a distributor instead of directly to end customers, the manufacturer should reduce both the length and size of the product line. This length adjustment is the opposite of what Villas-Boas (1996, 1998) established for the independent product setting. As noted by Villas-Boas (1998, p. 158), pricing distortion in an indirect channel increases "the cannibalization forces across the product line [and] the manufacturer tries to compensate for this by increasing the product differentiation across the line" (i.e., increasing the line length). In the independent product setting, qualities can be adjusted without affecting the quantities made available. That is not the case in a co-product setting because the quantities of the co-products are constrained by the output quality distribution and the overall quantity produced. The manufacturer cannot increase the product line length while maintaining the same quantity for each profitable market segment. Therefore, to deal with the reduced profit margin in the indirect channel, the co-product manufacturer should control the output quantities by reducing the initial production quantity and increasing the quality of the lowest grade (thus decreasing the product line length). This result implies that the quantity-related quality forces appear to dominate the cannibalization-related quality forces in a co-product setting. Additionally, we show that there exists a theoretical contract, combining revenue sharing and reverse slotting fees, that can eliminate the indirect channel distortions in both product line design and output quantities.

We compared a pure indirect channel to a pure direct channel. Oftentimes, a manufacturer may sell through both channels. Assuming it must offer the same product line to both channels, our results indicate that the indirect channel should exert a reducing-force on the length of the product line. One might anticipate that the optimal product line would more closely resemble the indirect (direct) channel optimal line if the indirect (direct) channel accounts for a larger fraction of the overall market. To resolve the distortions in product line design and output quantities, the coordinating contract discussed in section 5.3 might be used in the indirect channel. On the other hand, distributors are not always exclusive and customers might choose between channels. These two observations suggest that another interesting direction might be to explore the impact of competition within or between channels.

Acknowledgments

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Appendix A. Analysis of the Bilevel Program (5)

Proposition A1. *For any given Q, the optimal cutoffs θ_n^* in problem (5) must satisfy $\theta_{n+1}^* > \theta_n^*$ for all $n = 1, 2, \ldots, N$.*

All proofs of the results in this appendix are relegated to Appendix S1. Proposition A1 indicates that every grade, once specified, must be sold to a positive segment of customers in the equilibrium. Technically, Proposition A1 implies $\lambda_n = 0$ in problem (5) for $n = 1, 2, \ldots, N$. To cope with the remaining complicating constraints with Lagrangian multipliers, we note that problem (5) can be viewed as a linear program (LP) with respect to λ_0, $\boldsymbol{\mu}$ and $\boldsymbol{w}$ when all other decision variables are fixed at any feasible values. Moreover, it can be shown that the LP is optimized when $\lambda_0 = 0$ and $\mu_n = 0$ for all $n = 1, 2, \ldots, N$, and the optimal w_n's can be derived in closed form.

Proposition A2. *For any given Q, in the optimal solution to problem (5), it holds that $w_n^* - w_{n-1}^* = (2\theta_n^* - 1)(x_n^* - x_{n-1}^*)$ for $n = 1, 2, 3, \ldots, N$*

Proposition A2 characterizes the relations among the optimal wholesale prices, specification vector and market cutoffs for any given production quantity Q. Our problem is thus reduced to

$$\max_{\boldsymbol{\theta},\boldsymbol{x}} \quad R^I(Q, N, \boldsymbol{x}, \boldsymbol{\theta}) = \sum_{n=1}^{N} (1 - \theta_n)(2\theta_n - 1)(x_n - x_{n-1}) \tag{A1a}$$

$$\text{s.t. } 0 \le \theta_{n+1} - \theta_n \le a_n \text{ for } n = 1, 2, \ldots, N, \theta_1 \ge 0 \tag{A1b}$$

$$\underline{x} \le x_1 \le x_2 \le \cdots \le x_N \le \bar{x} \tag{A1c}$$

$$a_n = Q[F(x_{n+1}) - F(x_n)] \text{ for } n = 1, 2, \ldots, N. \tag{A1d}$$

Analyzing the above formulation, we are able to prove Lemma 2 in the main paper and obtain the simplified problem (7). The following proposition shows that for both indirect and direct channels and any given Q, x_n must be strictly greater than x_{n-1} for all n. Moreover, for each channel, there exists a lower bound for the quality level of the lowest grade.

Proposition A3. *For any given Q, the optimal specification vectors in the indirect and direct channels satisfy the following properties:*

(*i*) *For the indirect channel,* $x^I_{N^I}(Q) > x^I_{N^I-1}(Q) > \cdots > x^I_1(Q) \geq x^I_{min}(Q)$, *where* $x^I_{min}(Q) = \underline{x}$ *if* $Q \leq \frac{1}{4}$ *and* $x^I_{min}(Q) = F^{-1}(1 - \frac{1}{4Q})$ *otherwise;*
(*ii*) *for the direct channel,* $x^D_{N^D}(Q) > x^D_{N^D-1}(Q) > \cdots > x^D_1(Q) \geq x^D_{min}(Q)$, *where* $x^D_{min}(Q) = \underline{x}$ *if* $Q \leq \frac{1}{2}$ *and* $x^D_{min}(Q) = F^{-1}(1 - \frac{1}{2Q})$ *otherwise;*
(*iii*) $x^I_{min}(Q) \geq x^D_{min}(Q)$.

For any given Q, problem (7) can therefore be solved by the following shortest-path problem. Define $r^I(Q, z_1, z_2) = (1 - 2Q\bar{F}(z_2))(z_2 - z_1)Q\bar{F}(z_2)$, and we have $R^I(Q, N, \boldsymbol{x}) = \sum_{n=1}^N r^I(Q, x_n, x_{n-1})$. Recall that in Problem 1, we aim to maximize $R^I(Q, N, \boldsymbol{x}) - C_p(Q) - C_b(Q, N)$. For any given Q, the optimization over N and $\boldsymbol{x}$ reduces to maximizing $\sum_{n=1}^N r^I(Q, x_n, x_{n-1}) - b_2(N-1)$. To convert this maximization problem to a shortest path problem, we discretize the interval $(x^I_{min}(Q), \bar{x})$ as $T+1$ points and choose the grade specifications from these points. T is set to be a sufficiently large number in order to obtain a precise solution. We can create a directed graph with the following setup.

- Create $T+2$ nodes, labeled $i = 0, \ldots, T+1$ with a directed arc (i, j) for every pair $i < j$.
- Set $x(0) = 0$ and $x(i) = x^I_{min}(Q) + (i-1)(\bar{x} - x_{min}(Q))/T$ for $i = 1, 2, \ldots, T+1$.
- The distance of arc (i, j) is given by $d(i,j) = \begin{cases} -r^I(Q, x(i), x(j)) \text{ for } i = 0 \\ -r^I(Q, x(i), x(j)) + b_2 \text{ for } i > 0 \end{cases}$

Note that a path from node 0 to $T+1$ in the above graph indicates a specification vector with the quality levels corresponding to the $x(i)$'s along the path. The absolute value of the total distance of a path from node 0 to $T+1$ is equal to the profit generated by the corresponding specification vector minus a constant b_2. Therefore, determining the optimal N and $\boldsymbol{x}$ is equivalent to finding the shortest path in the above graph. For general problems, the shortest-path algorithm needs to be run for each possible Q in order to determine the optimal production quantity Q^I, $\boldsymbol{x}^I$ and N^I for the manufacturer in the indirect channel. The direct-channel case can be solved in the same fashion except that the revenue function has a different expression.

Lemma A1. *Let* $\boldsymbol{x}^I(Q, N)$ *denote the manufacturer's optimal specification vector that maximizes* $R^I(Q, N, \boldsymbol{x}) = \sum_{n=1}^N r^I(Q, x_n, x_{n-1})$ *for any given* Q *and* N. *The revenue function* $R^I(Q, \boldsymbol{x}^I(Q, N))$ *is increasing in* N.

Lemma A1 implies that it is optimal to set infinitely many grades when $b_2 = 0$. For this special case, the following proposition characterizes the optimal specification vectors and the resulting revenue functions for any given Q in both indirect and direct channels.

Proposition A4. *When* $b_2 = 0$, *for any given* Q, (*i*) $N^{*I} = N^{*D} = \infty$, (*ii*) $x^{*I}_1 = x^I_{min}(Q)$, $x^{*D}_1 = x^D_{min}(Q)$ *and* $x^{*I}_N = x^{*D}_N = \bar{x}$, *and* (*iii*) *the revenue functions for the indirect and direct channels,* $\hat{R}^I(Q)$ *and* $\hat{R}^D(Q)$, *are strictly concave in* Q *and can be written as*

$$\hat{R}^I(Q) = \begin{cases} \underline{x}Q(1-2Q) + Q\int_{\underline{x}}^{\bar{x}}[1-2Q\bar{F}(z)]\bar{F}(z)dz & \text{if } Q \leq \frac{1}{4} \\ \frac{1}{8}F^{-1}(1-\frac{1}{4Q}) + Q\int_{F^{-1}(1-\frac{1}{4Q})}^{\bar{x}}[1-2Q\bar{F}(z)]\bar{F}(z)dz & \text{if } Q > \frac{1}{4}, \end{cases} \quad \text{(A2)}$$

and

$$\hat{R}^D(Q) = \begin{cases} \underline{x}Q(1-Q) + Q\int_{\underline{x}}^{\bar{x}}[1-Q\bar{F}(z)]\bar{F}(z)dz & \text{if } Q \leq \frac{1}{2} \\ \frac{1}{4}F^{-1}(1-\frac{1}{2Q}) + Q\int_{F^{-1}(1-\frac{1}{2Q})}^{\bar{x}}[1-Q\bar{F}(z)]\bar{F}(z)dz & \text{if } Q > \frac{1}{2}, \end{cases} \quad \text{(A3)}$$

Corollary A1. *Assuming* $C_p(Q) = c_1 Q + c_2 Q^2$ *and* $F(\cdot) \sim U(\underline{x}, \bar{x})$, *if* $b_2 = 0$ *so that complete classification is optimal for both settings, the unique optimal production quantity and product line design are characterized as follows.*

(*i*) *Indirect channel:*

$$Q^{*I} = \begin{cases} \frac{[\mu - b_1 - c_1]^+}{4(\mu - \sigma/\sqrt{3}) + 2c_2} & \text{if } b_1 + c_1 + \frac{c_2}{2} \geq \frac{\sigma}{\sqrt{3}}; \\ \text{the unique solution to } 2c_2 Q^3 + (b_1 + c_1)Q^2 = \frac{\sigma}{16\sqrt{3}} & \text{otherwise;} \end{cases}$$

$x^{*I}_1 = x^I_{min}(Q^{*I})$ *and* $x^{*I}_N = \mu + \sqrt{3}\sigma$.

(*ii*) *Direct channel:*

$$Q^{*D} = \begin{cases} \frac{[\mu - b_1 - c_1]^+}{2(\mu - \sigma/\sqrt{3} + c_2)} & \text{if } b_1 + c_1 + c_2 \geq \frac{\sigma}{\sqrt{3}}; \\ \text{the unique solution to } 2c_2 Q^3 + (b_1 + c_1)Q^2 = \frac{\sigma}{4\sqrt{3}} & \text{otherwise;} \end{cases}$$

$x^{*D}_1 = x^D_{min}(Q^{*D})$ *and* $x^{*D}_N = \mu + \sqrt{3}\sigma$.

Proposition A5. *Assuming* $C_b(Q, N) + C_p(Q) = cQ^2$, *in the indirect-channel setting, the manufacturer's and the distributor's profits are given by*

$$\Pi^{*I}_m = \begin{cases} \frac{\mu^2}{4(c + 2\mu - 2\sigma/\sqrt{3})} & \text{if } c \geq \frac{2\sigma}{\sqrt{3}}, \\ \frac{1}{8}\left(\mu + \sqrt{3}\sigma - \frac{1}{2}(6\sigma)^{2/3}c^{1/3}\right) & \text{if } c < \frac{2\sigma}{\sqrt{3}}, \end{cases}$$

$$\Pi^{*I}_d = \begin{cases} \frac{\mu^2(\mu - \sigma/\sqrt{3})}{4(c + 2\mu - 2\sigma/\sqrt{3})^2} & \text{if } c \geq \frac{2\sigma}{\sqrt{3}}, \\ \frac{1}{16}\left(\mu + \sqrt{3}\sigma - \frac{2}{3}(6\sigma)^{2/3}c^{1/3}\right) & \text{if } c < \frac{2\sigma}{\sqrt{3}}. \end{cases}$$

In the direct-channel setting, the manufacturer's profit is given by

$$\Pi_m^{*D} = \begin{cases} \frac{\mu^2}{4(c+\mu-\sigma/\sqrt{3})} \text{ if } c \geq \frac{\sigma}{\sqrt{3}}, \\ \frac{1}{4}\left(\mu + \sqrt{3}\sigma - (3\sigma)^{2/3}c^{1/3}\right) \text{ if } c < \frac{\sigma}{\sqrt{3}}. \end{cases}$$

Notes

[1]This 56% is the percentage of Cree's total revenue over all product segments, rather than the exact proportion of revenue from LEDs.

[2]See, for example, http://www.e6.com/en/Home/Contact+us/Asia+Pacific/.

[3]Although manufacturers often sell through both indirect and direct channels, in this study, we compare a pure indirect channel with a pure direct channel to isolate and highlight the impact of an indirect channel.

[4]Throughout this study, we follow the terminology used in Netessine and Taylor (2007) and Chen et al. (2013), for example, in which length refers to the quality difference between the highest and lowest products. This terminology varies somewhat across product-line papers, with some, e.g., Dong et al. (2018), using length to refer to the number of products (size in our terminology).

[5]In the independent product setting, e.g., Villas-Boas (1998), quality is costly in the sense that the marginal production cost of a product is an increasing function of product quality. For our co-product setting, the marginal cost to manufacture grade n is $(c + b_1)/[F(x_{n+1}) - F(x_n)]$ in the linear production cost case $C_p(Q) = cQ$. The marginal cost to manufacture grades of quality x_n or higher is $(c + b_1)/[1 - F(x_n)]$ which is increasing in x_n. Therefore, it is more expensive to generate grades n and higher as the quality of grade n increases. In this sense, quality is also costly in a co-product setting.

[6]The assumption of uniform distribution is somewhat common in the literature; for example, Villas-Boas (1996), Shi et al. (2013), and Jerath et al. (2017). However, we relax the assumption in section 5.1 and establish robustness of our key findings.

[7]As in Chen et al. (2013), customers evaluate the quality of grade n as x_n, that is, the lowest quality in its interval.

[8]To guarantee that KKT conditions are necessary and sufficient for optimality, we need to check if the constraint qualification holds in problem (4). Following section 11 of Luenberger and Ye (1984), it suffices to check if at the optimal $\boldsymbol{\theta}$, the gradient vectors of all active constraints in (4) are linearly independent. Note that some of these gradient vectors could be linearly dependent only if constraints $\theta_{n+1} - \theta_n \leq a_n$ and $\theta_{n+1} - \theta_n \geq 0$ are both active for some n. This could happen only when $a_n = 0$, implying $x_{n+1} = x_n$. However, this cannot be an equilibrium outcome as the manufacturer in this case can always reduce the number of grades from N to $N - 1$ to save the classification cost $C_b(Q, N)$.

[9]In fact, the lost margin $p_n^* - w_n^* = \sum_{k=1}^{n}(x_k^* - x_{k-1}^*)Q\bar{F}(x_k^*)$ is increasing in n, and so double marginalization (in an absolute sense) is greatest for the highest quality grade offered. Thus, the manufacturer's incentive to reduce the sales volume of grade N is stronger than that of any other grade.

[10]We numerically observed that the property stated in Proposition 2 holds for other grades as well, that is, for each $n = 1, 2, \ldots, N - 1$, $x_n^{*I} \geq x_n^{*D}$ when all the other decisions are fixed. The corresponding numerical study is reported in section S.4 of Appendix S1.

[11]Corollary A1 in the Appendix characterizes the production quantities and product line designs when $C_p(Q) + C_b(Q, N)$ is a general quadratic function of Q.

[12]We note that the relative reduction becomes larger as b_2 increases over the range shown. We also tested a number of instances with even larger b_2 values and found that the product line relative reduction grows more slowly in b_2 until the length metric becomes ill-defined at high enough values of b_2 that induce the separation (single-grade) strategy in which case length is not a meaningful metric.

[13]In the other three instances there was no change in the quality of the lowest grade.

[14]To compute the value of $\hat{\Pi}_m^{*I}$ for each instance, we solved problem (A1) in the Appendix with the product line design fixed as $N = N^{*D}$ and $\boldsymbol{x} = \boldsymbol{x}^{*D}$. To be consistent with previous studies, the optimal Q is found by an exhaustive search with the same discretization.

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Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Appendix S1. Supplementary Material.