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On the Value of Mix Flexibility and Dual Sourcing in Unreliable Newsvendor Networks

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We connect the mix-flexibility and dual-sourcing literatures by studying unreliable supply chains that produce multiple products. We consider a firm that can invest in product-dedicated resources and totally flexible resources. Product demands are uncertain at the time of resource investment, and the products can differ in their contribution margins. Resource investments can fail, and the firm may choose to invest in multiple resources for a given product to mitigate such failures.

In comparing a single-source dedicated strategy with a single-source flexible strategy, we refine the common intuition that a flexible strategy is strictly preferred to a dedicated strategy when the dedicated resources are costlier than the flexible resource. We prove that this intuition is correct if the firm is risk neutral or if the resource investments are perfectly reliable. The intuition can be wrong, however, if both of these conditions fail to hold, because there is a resource-aggregation disadvantage to the flexible strategy that can dominate the demand pooling and contribution-margin benefits of the flexible strategy when resource investments are unreliable and the firm is risk averse.

We investigate the influence that resource attributes, firm attributes, and product-portfolio attributes have on the attractiveness of various supply-chain structures that differ in their levels of mix flexibility and diversification, and we investigate the influence these attributes have on the optimal resource investments within a given supply-chain structure. Our results indicate that the appropriate levels of diversification and flexibility are very sensitive to the resource costs and reliabilities, the firm's downside risk tolerance, the number of products, the product demand correlations and the spread in product contribution margins.

Key words: reliability; flexibility; dual sourcing; loss aversion; risk

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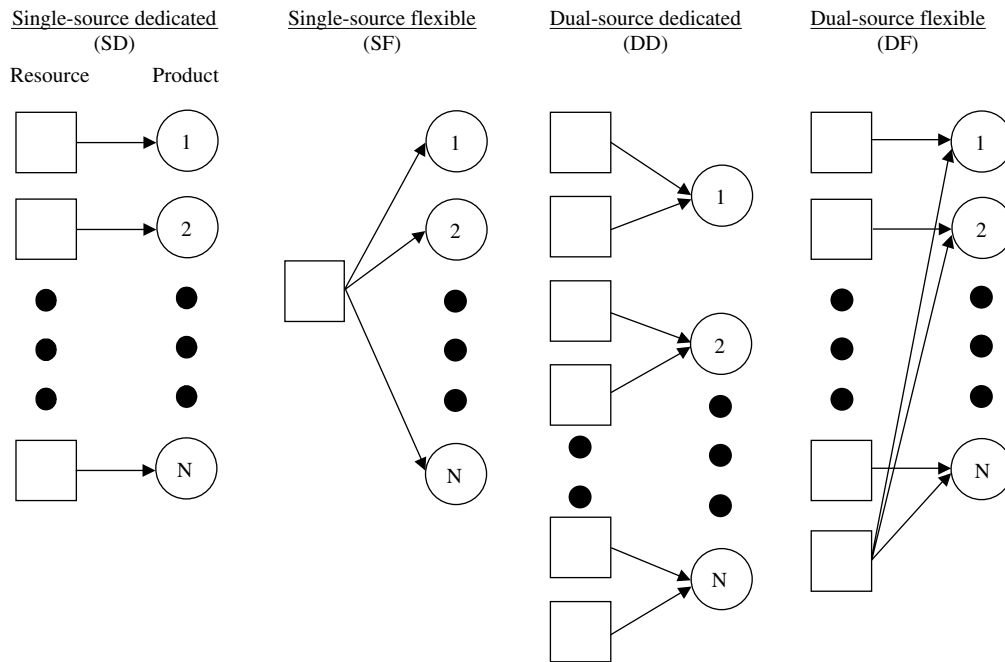
1. Introduction

It is well established, both in the literature and in practice, that resource flexibility is advantageous for firms that sell multiple products with uncertain demand, and that dual sourcing is advantageous for firms that face uncertainty in supply. Research to date, however, has studied these supply-chain strategies in isolation: The mix-flexibility literature has assumed perfectly reliable supply, and the dual-sourcing literature has focused on single-product problems.

Consider a firm selling multiple products that differ in their contribution margins (sales price less variable costs) and have uncertain demands. The firm might invest in product-dedicated resources only, or, alternatively, it might invest in one single flexible resource that can produce all products. The dedicated resources might be cheaper but the flexible strategy

offers a demand-pooling benefit and a contribution-margin option benefit (Van Mieghem 1998). There is, however, a resource-aggregation disadvantage to the flexible strategy that to the best of our knowledge has been ignored in the mix-flexibility literature. A single failure in the flexible strategy leaves the firm with no productive resource, whereas with the dedicated strategy all resources must fail for the firm to be in similar straits. Supply uncertainty, by which we mean that the realized resource investment may differ from the target investment, should therefore influence the firm's preference for either a dedicated or flexible strategy. The firm, of course, is not limited to these two strategies, and it may want to consider dual sourcing for one or more products to protect itself against supply uncertainties. The goal of this paper is to simultaneously study mix flexibility and dual

Figure 1 Four Network Structures



sourcing to provide insight into effective supply-chain design in the presence of supply and demand uncertainties.

Figure 1 illustrates four canonical network structures for a firm that sells N products. Circles represent products, squares represent resources, and arcs represent the ability of a resource to fulfill demand for a product. SD represents a single-source dedicated network, SF represents a single-source flexible network, DD represents a dual-source dedicated network, and DF represents a dual source flexible network. We note that these structures represent possible sourcing decisions rather than actual decisions, and so a firm with a DD or a DF network may choose to single source one or more products, even though it could dual source them.

The dual-sourcing networks (DD and DF) offer diversification benefits that are advantageous in the presence of unreliable resource investments. The flexible strategies (SF and DF) offer demand-pooling and contribution-margin benefits that are advantageous in the presence of demand uncertainty. The demand-pooling benefit arises only if demands are not perfectly positively correlated. The contribution-margin benefit arises only if product contribution margins

differ. In the presence of supply uncertainty, there is a resource-aggregation disadvantage to a flexible resource. In general, the desirability of any of the four networks will be influenced by resource investment costs, resource reliabilities, product contribution margins, demand correlations, and the firm’s attitudes toward risk.

Mix flexibility, whereby a resource has the ability to produce multiple products, has been investigated in the operations literature as a design strategy for firms that sell multiple products with uncertain demand. Hereafter, we will simply use the term flexibility rather than mix flexibility. Such literature has primarily focused on single-period (newsvendor type) investments in dedicated and totally flexible resources, and that is the focus of this paper. We refer the reader to Jordan and Graves (1995), Graves and Tomlin (2003), and Muriel et al. (2004) for treatments of partial flexibility. For single-period investments in dedicated and totally flexible resources, Van Mieghem (2004a) establishes that component commonality and resource flexibility are distinctions without a difference; the problems can be shown to be mathematically equivalent. As such, we use the term *resource* with the understanding that the resource in question

might be inventory or capacity. The resource may be produced in house or may be provided by an outside supplier.

The archetypal total-flexibility model (e.g., Fine and Freund 1990) is a single-period, N product, uncertain-demand model in which a risk-neutral firm can invest in N dedicated resources and one totally flexible resource. Investment costs are linear and there are no fixed costs. Gupta et al. (1992), Li and Tirupati (1994, 1995, 1997), and Van Mieghem (1998, 2004b) all investigate variations on this theme. The paper of most direct relevance is Van Mieghem (1998). In that model, the firm sells two products that differ in contribution margins. This difference in contribution margins makes flexibility valuable even if demands for both products are perfectly positively correlated, a result that contradicted the prevailing intuition. Our work relaxes two implicit assumptions (reliable investments and risk neutrality) of these models. There is a burgeoning literature on non-risk-neutral decision makers in the single-product newsvendor context, e.g., Eeckhoudt et al. (1995), Agrawal and Seshadri (2000), Schweitzer and Cachon (2000), Caldentey and Haugh (2004), and Chen et al. (2003). As far as we are aware, the only mix-flexibility paper (other than this one) to relax the risk-neutrality assumption is Van Mieghem (2004b). That paper and our paper can be seen as complementary, in that the research questions being addressed differ, as do the treatments of non-risk-neutral decision makers. Van Mieghem (2004b) investigates how risk aversion influences the flexibility investment levels in perfectly reliable newsvendor networks by using concave-increasing utility functions (to investigate the directional influence of risk aversion) and a mean-variance approach (to investigate the magnitude of the influence). In contrast, we investigate flexibility and dual sourcing in unreliable newsvendor networks and, in doing so, allow for non-risk-neutral firms by considering both loss aversion (Kahneman and Tversky 1979) and the Conditional Value-at-Risk (CVaR) measure (Rockafellar and Uryasev 2000, 2002).

Unreliable, single-product, single-resource problems have been widely studied in the yield and disruption literatures. In contrast, unreliable supply chains (multiple resources or multiple products, or both) have received less attention. Dual-sourcing

strategies have been investigated in the context of random yield (Gerchak and Parlar 1990, Parlar and Wang 1993, Anupindi and Akella 1993, Agrawal and Nahmias 1997, Swaminathan and Shanthikumar 1999, Dada et al. 2003, Tomlin 2004b), random disruptions (Parlar and Perry 1996, Gürler and Parlar 1997, Tomlin 2004a), and credit risk (Babich et al. 2004), but all these papers assume a single product, so mix flexibility is not relevant. We note that Tomlin (2004a) investigates the value of volume flexibility in unreliable supply chains.

The rest of the paper is organized as follows. Section 2 introduces the general supply chain model. In §3 we consider the SD and SF networks. In §4 we consider the DF and DD networks, and compare them to the single-source networks. Conclusions and directions for future research are presented in §5. Proofs of all results can be found in Appendix A.

2. The Model

We present a general model and identify each of the supply networks (SD, SF, DD, and DF) as instances of the general model. There are N products $n = 1, \dots, N$. The marginal contribution margin for product n is p_n . We use the notational convention that $p_1 \geq p_2 \geq \dots \geq p_N$. Let $\mathbf{p} = (p_1, \dots, p_N)$. All vectors are assumed to be column vectors, and $'$ denotes the transpose operator. The firm can invest in nonnegative levels (K_j) of J different resources labeled $j = 1, \dots, J$. Let \mathbf{T} be the $N \times J$ technology matrix with $t_{nj} = 1$, indicating that resource j can produce product n . Demand $\tilde{\mathbf{X}} = (\tilde{X}_1, \dots, \tilde{X}_N)$ is uncertain, with a joint density $f_{\mathbf{X}}(x_1, \dots, x_N)$ at the time of the investment decision. The demand-correlation matrix is denoted $\boldsymbol{\rho}_{\mathbf{X}}$ with element ρ_{mn} being the correlation coefficient for products m and n . The marginal density for \tilde{X}_n is $f_{X_n}(x_n)$ and the cumulative distribution function is $F_{X_n}[x_n]$. Realizations of demand are denoted $\mathbf{x} = (x_1, \dots, x_N)$.

Resource investments are unreliable and the realized level \tilde{K}_j^r for resource j is stochastically proportional to the invested level K_j , i.e., $\tilde{K}_j^r = \tilde{Y}_j K_j$. In particular, we assume a Bernoulli yield model in which $\tilde{Y}_j = 1$ with probability θ_j and $\tilde{Y}_j = 0$ with probability $1 - \theta_j$. We refer to θ_j as the reliability of resource j . There is often a Bernoulli nature to the supply process (e.g., Anupindi and Akella 1993, Parlar et al. 1995, Swaminathan and Shanthikumar 1999, Dada et al.

2003, Tomlin 2004b). In the case of inventory, Bernoulli supply processes arise due to batch failures, acceptance sampling, supply-chain disruptions, or supplier delays (if the delivery occurs too late to serve demand). Unless otherwise stated, we assume that the \tilde{Y}_j are independent. Let $\tilde{\mathbf{Y}} = (\tilde{Y}_1, \dots, \tilde{Y}_J)$ and let a realization be denoted by $\mathbf{y} = (y_1, \dots, y_J)$.

Investment costs for an unreliable resource can depend on both the investment (or ordered) level and the realized (or delivered) level. For each resource j , we assume that the firm pays $\lambda_j c_j$ per unit ordered and an additional $(1 - \lambda_j)c_j$ per unit delivered. We refer to c_j as the marginal total cost and λ_j as the marginal committed cost. Equivalently, we can think of the firm paying c_j per unit ordered but receiving a rebate of $(1 - \lambda_j)c_j$ for each unit not delivered; in that context $(1 - \lambda_j)$ is the failure rebate. Let $\mathbf{c} = (c_1, \dots, c_J)$, $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_J)$, $c_j(y_j) = c_j(\lambda_j + (1 - \lambda_j)y_j)$, and $\mathbf{c}(\mathbf{y}) = (c_1(y_1), \dots, c_J(y_J))$.

The firm's investment problem can be formulated as a two-stage stochastic program. In the second stage, after demands and investments have been realized, the firm allocates production to maximize the contribution.

$$r(\mathbf{K}, \mathbf{x}, \mathbf{y}) = \max_{s_n, q_{nj} \geq 0} \mathbf{p}'\mathbf{s} \quad (1)$$

$$\text{s.t.} \quad s_n \leq x_n, \quad n = 1, \dots, N \quad (2)$$

$$s_n \leq \sum_{j=1}^J t_{nj} q_{nj}, \quad n = 1, \dots, N \quad (3)$$

$$\sum_{n=1}^N q_{nj} \leq y_j K_j, \quad j = 1, \dots, J, \quad (4)$$

where q_{nj} denotes the production of product n by resource j , $\mathbf{s} = (s_1, \dots, s_N)$ denotes the sales of products $1, \dots, N$, constraint (2) ensures that sales do not exceed realized demand, constraint (3) ensures that sales do not exceed production, and constraint (4) ensures that resource usage does not exceed the realized level.

Let w_0 be the firm's initial wealth and let $\tilde{W}(\mathbf{K})$ be the random gain or loss (denoted by positive or negative numbers, respectively) achieved by investment \mathbf{K} . The firm's realized profit on investment \mathbf{K} is given by $w(\mathbf{K}) = -\mathbf{c}(\mathbf{y})'\mathbf{K} + r(\mathbf{K}, \mathbf{x}, \mathbf{y})$. The firm's random terminal wealth is then $w_0 + \tilde{W}(\mathbf{K})$. In the first stage, before demands and yields are realized, the firm chooses

a nonnegative investment vector $\mathbf{K} = (K_1, \dots, K_J)$ to maximize some objective function $V(\mathbf{K})$, where $V(\mathbf{K})$ depends on the firm's terminal wealth. We consider three different types of firms: a risk-neutral firm, a loss-averse firm, and a firm concerned about downside risk, each represented by a distinct form of the objective function.

The risk-neutral objective function is given by $V_{\text{RN}}(\mathbf{K}) = w_0 + E_{\tilde{\mathbf{x}}, \tilde{\mathbf{y}}}[\tilde{W}(\mathbf{K})]$, so the risk-neutral investment problem is

$$V_{\text{RN}}(\mathbf{K}^*) = w_0 + \max_{\mathbf{K} \geq 0} E_{\tilde{\mathbf{x}}, \tilde{\mathbf{y}}}[\tilde{W}(\mathbf{K})]. \quad (5)$$

A loss-averse decision maker (Kahneman and Tversky 1979) attributes more significance to losses than to gains. Schweitzer and Cachon (2000) study loss aversion in a classic single-product newsvendor setting by using a piecewise-linear model that is a special case of Tversky and Kahneman's (1992) two-part power-function model. We assume the same piecewise linear model here; $V_{\text{LA}}(\mathbf{K}) = w_0 + E_{\tilde{\mathbf{x}}, \tilde{\mathbf{y}}}[\tilde{W}^+(\mathbf{K}) - \beta \tilde{W}^-(\mathbf{K})]$ where $\tilde{W}^+(\mathbf{K}) = \max\{\tilde{W}(\mathbf{K}), 0\}$, $\tilde{W}^-(\mathbf{K}) = \max\{-\tilde{W}(\mathbf{K}), 0\}$ and $\beta \geq 1$. Increasing loss aversion is associated with an increasing β . We note that $V_{\text{LA}}(\mathbf{K}) = V_{\text{RN}}(\mathbf{K})$ at $\beta = 1$. The loss-averse investment problem is

$$V_{\text{LA}}(\mathbf{K}^*) = w_0 + \max_{\mathbf{K} \geq 0} E_{\tilde{\mathbf{x}}, \tilde{\mathbf{y}}}[\tilde{W}^+(\mathbf{K}) - \beta \tilde{W}^-(\mathbf{K})]. \quad (6)$$

For the terminal wealth distribution associated with a resource-investment vector \mathbf{K} , the CVaR, denoted $V_{\text{CVaR}_\eta}(\mathbf{K})$, is the mean of the left η -tail of the wealth distribution. The percentile $\eta \in (0, 1)$ is a parameter that reflects the firm's taste for downside risk. At $\eta = 1$, the firm is risk neutral, and the complete wealth distribution is considered in the objective. For $\eta < 1$, the firm maximizes the mean of the wealth distribution falling below a specified percentile level η . Increasing concern with downside risk is associated with a decreasing percentile η . In recent years, the CVaR measure has gained popularity as a risk measure in the finance literature, e.g., Rockafellar and Uryasev (2000, 2002), Acerbi (2002), Acerbi and Tasche (2002), and Szegö (2002). Using Theorem 10 of Rockafellar and Uryasev (2002),

$$V_{\text{CVaR}_\eta}(\mathbf{K}) = w_0 + \max_v \left\{ v + \frac{1}{\eta} E_{\tilde{\mathbf{x}}, \tilde{\mathbf{y}}}[\min\{\tilde{W}(\mathbf{K}) - v, 0\}] \right\}, \quad (7)$$

i.e., for a given investment vector \mathbf{K} , $V_{\text{CVaR}_\eta}(\mathbf{K})$ can be found by solving a maximization problem. We can therefore write the CVaR investment problem as

$$V_{\text{CVaR}_\eta}(\mathbf{K}^*) = w_0 + \max_{\mathbf{K} \geq 0, v} \left\{ v + \frac{1}{\eta} E_{\tilde{X}, \tilde{Y}}[\min\{\tilde{W}(\mathbf{K}) - v, 0\}] \right\}. \quad (8)$$

$V_{\text{CVaR}_\eta}(\mathbf{K})$ is jointly concave over $\mathbf{K} \geq 0$ and v ; see Rockafellar and Uryasev (2000, 2002). It is this joint concavity property, in addition to the fact that $V_{\text{CVaR}_\eta}(\mathbf{K})$ is a coherent measure of risk as defined in Artzner et al. (1999), that has given rise to its popularity. The optimal investment is independent of the initial wealth for all three objective functions, and we therefore assume that $w_0 = 0$ without loss of generality.

In closing, we note that each of the four supply networks (SD, SF, DD, and DF) can be obtained as a special case of the general model presented above. For example, if $N = 2$, then

$$\begin{aligned} \text{SD: } T &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; & \text{SF: } T &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \\ \text{DD: } T &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}; & \text{DF: } T &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}. \end{aligned}$$

3. Single-Source Networks: The Flexibility Premium

In this section, we focus on the single-source networks SD and SF. There are N resources in the SD network, and we label these $n = 1, \dots, N$ with resource n dedicated to product n . There is a single (flexible) resource in SF, and we label this $N + 1$. We focus on the counterbalancing effects of demand pooling and resource aggregation by assuming $p_1 = p_2 = \dots = p_N$ to eliminate the contribution-option benefit. By eliminating this contribution-option benefit, we can model the SF investment problem as a single-product problem with demand $\tilde{X}_{N+1} = \tilde{X}_1 + \tilde{X}_2 + \dots + \tilde{X}_N$. We denote the density and cumulative distribution of total demand by $f_{X_{N+1}}(x_{N+1})$ and $F_{X_{N+1}}[x_{N+1}]$, respectively.

Let $V^{\text{SD},*}$ and $V^{\text{SF},*}$ denote the optimal objective values for the SD and SF networks. SF is strictly preferred if $V^{\text{SF},*} > V^{\text{SD},*}$, and is weakly preferred if $V^{\text{SF},*} \geq V^{\text{SD},*}$. Hereafter, the term *preferred* should be understood to mean weakly preferred. Clearly the firm's preference will depend on the resource costs and the reliabilities, so neither network will dominate

the other in the sense that it is preferred for all possible parameters. To gain insight into what drives a firm's network preference, we restrict attention in this section to the special case where the following three assumptions all hold.

1. The resource reliabilities are identical for all resources, i.e., $\theta_1 = \theta_2 = \dots = \theta_N = \theta_{N+1} = \theta$, but the yield random variables are still independent across resources.

2. The marginal committed costs are identical for all resources, i.e., $\lambda_1 = \lambda_2 = \dots = \lambda_N = \lambda_{N+1} = \lambda$.

3. The marginal total costs are identical for the dedicated resources, i.e., $c_1 = c_2 = \dots = c_N = c$.

We now introduce two definitions, the second of which will be a key metric in much of the analysis.

DEFINITION 1. The *indifference cost* c_{N+1}^l is the value of the flexible resource's marginal total cost at which the firm is indifferent between the SD and SF networks; that is, c_{N+1}^l is the value of c_{N+1} such that $V^{\text{SF},*} = V^{\text{SD},*}$.

DEFINITION 2. The *flexibility premium* Δ is the relative difference in the marginal total costs at which the firm is indifferent between the SD and SF networks, i.e., $\Delta = (c_{N+1}^l - c)/c$.

The flexibility premium Δ is a useful measure of the value of flexibility; the firm prefers the SF network as long as $c_{N+1} \leq (1 + \Delta)c$. Put another way, $\Delta > 0$ implies that the firm is willing to pay a higher price (relative to the dedicated resources) for flexibility, whereas $\Delta < 0$ implies that the firm requires a lower price for flexibility to be preferred.

We note that although we assume that resource reliabilities, marginal committed costs, and dedicated marginal total costs are identical in this section, many of the following equations ((9)–(24)) can be extended to the nonidentical case in a straightforward manner.

3.1. Risk-Neutral Firm

This SF investment problem is an extension of the classic single-product newsvendor model to allow for Bernoulli investment failures and the specified investment costs. The objective function is

$$\begin{aligned} V_{\text{RN}}^{\text{SF}}(K_{N+1}) &= -(\lambda + (1 - \lambda)\theta)c_{N+1}K_{N+1} \\ &+ \theta p \left[\int_0^{K_{N+1}} x_{N+1} f_{X_{N+1}}(x_{N+1}) dx_{N+1} \right. \\ &\left. + K_{N+1}(1 - F_{X_{N+1}}[K_{N+1}]) \right]. \quad (9) \end{aligned}$$

It is relatively straightforward to show that

$$K_{N+1, RN}^* = F_{X_{N+1}}^{-1} \left[1 - \frac{(\lambda + (1 - \lambda)\theta)c_{N+1}}{\theta p} \right] \quad (10)$$

and

$$V_{RN}^{SF,*} = \theta p \int_0^{K_{N+1}^*} x_{N+1} f_{X_{N+1}}(x_{N+1}) dx_{N+1}. \quad (11)$$

The SD investment problem can be modeled as N independent single-product problems, so

$$K_{n, RN}^* = F_{X_n}^{-1} \left[1 - \frac{(\lambda + (1 - \lambda)\theta)c}{\theta p} \right], \quad n = 1, \dots, N \quad (12)$$

and

$$V_{RN}^{SD,*} = \theta p \sum_{n=1}^N \int_0^{K_n^*} x_n f_{X_n}(x_n) dx_n. \quad (13)$$

For the case of $N = 2$ (i.e., two products), the risk-neutral flexibility premium Δ_{RN} is plotted as a function of both θ and λ in Figures 2 and 3 for independent uniformly distributed $U(0, 1)$ demands.

We see that Δ_{RN} can be increasing or decreasing in both θ and λ , depending on the magnitude of p/c . Observe that Δ_{RN} is constant for $\lambda = 0$ in both cases. This observation is true in general, because the optimal resource investment is independent of the reliability ($K_{n, RN}^* = F_{X_n}^{-1}[1 - c/p]$) if investment failures are fully rebated, i.e., $\lambda = 0$. Observe that the flexibility premium is nonnegative for all (θ, λ) combinations in

Figure 2 Flexibility Premium for Risk-Neutral Objective with $p/c = 2$

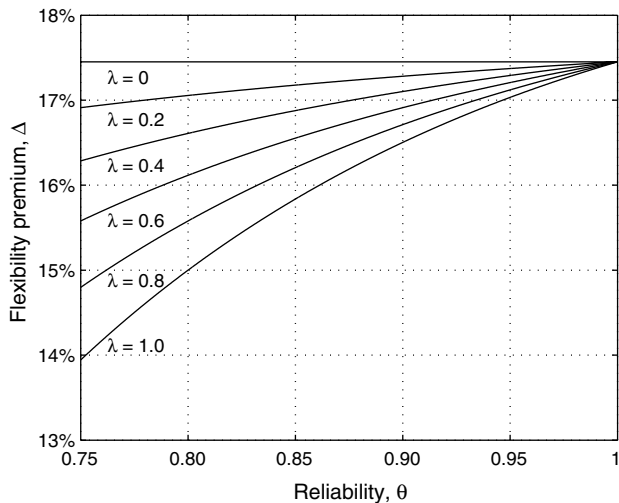
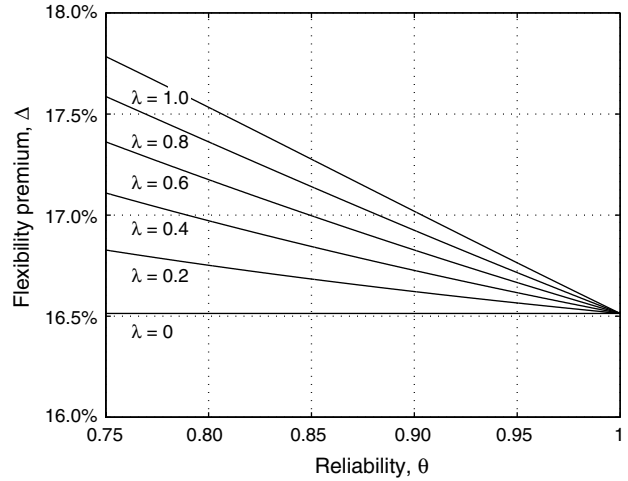


Figure 3 Flexibility Premium for Risk-Neutral Objective with $p/c = 5$



both figures; that is, SF is always preferred if $c_{N+1} \leq c$. In fact, as the following proposition shows, this observation is true in general.

PROPOSITION 1. For any demand random vector $\tilde{\mathbf{X}}$ (i) the risk-neutral flexibility premium Δ_{RN} is nonnegative for all $0 \leq \theta \leq 1$, (ii) $0 \leq \theta \leq \lambda c / (p - (1 - \lambda)c) \Rightarrow \Delta_{RN} = 0$, and (iii) $\Delta_{RN} = 0$ if $\mathbf{p}_X = \mathbf{1}$, i.e., pairwise perfect positive correlation for all products.

This proposition tells us that the SF network is preferred to the SD network for all reliabilities and for all demand distributions in the case of a risk-neutral firm facing equal resource reliabilities and costs. This result was not obvious a priori (at least to the authors), because the demand-pooling benefit of SF might be outweighed by the resource-aggregation disadvantage. In fact, even when there is no demand-pooling benefit (i.e., $\mathbf{p}_X = \mathbf{1}$), the firm is still indifferent between SF and SD. Recall that the contribution-margin benefit does not exist because $p_1 = p_2 = \dots = p_N$. The only explanation is that resource aggregation does not exclusively penalize SF. Resource aggregation is a disadvantage for SF from a downside perspective because the probability of multiple resources failing in SD is less than that of the single resource failing in SF, but an advantage from an upside perspective because the probability of the single resource succeeding in SF is higher than the probability of multiple resources succeeding in SD. For a risk-neutral firm, the upside resource-disaggregation advantage, coupled with the demand-pooling benefit of the SF network, dominates

the downside resource-disaggregation disadvantage, resulting in SF being preferred to SD. Allowing for asymmetric contribution margins only increases the advantage of the SF network.

Risk neutrality implies a utility function that has a constant marginal return to wealth. Resource aggregation influences the distribution of the terminal wealth, so it is natural to ask whether alternate wealth preferences would change the result that SF is preferred to SD. We now address this question by considering the loss-averse and CVaR objective functions.

3.2. Loss-Averse Firm

We assume that $N = 2$ in this section, but the results for the SF network extend in an obvious fashion to the case where there are more than two products. Recall that, using our convention, the total demand is labeled \tilde{X}_3 and the flexible resource is labeled K_3 when $N = 2$ (because $N + 1 = 3$).

The SF investment problem is an extension of the loss-averse single-product newsvendor model to allow for Bernoulli investment failures and the specified investment costs. The objective function is

$$V_{LA}^{SF}(K_3) = E_{X_3, Y_3}[\tilde{W}_3(K_3)] + (\beta - 1)E_{X_3, Y_3}[\tilde{W}_3(K_3) | \tilde{W}_3(K_3) < 0], \quad (14)$$

where $\tilde{W}_3(K_3)$ is the random profit realized by an investment of K_3 . Therefore,

$$V_{LA}^{SF}(K_3) = -(\lambda + (1 - \lambda)\theta)c_3K_3 + \theta p \left(\int_0^{K_3} x_3 f_{X_3}(x_3) dx_3 + K_3(1 - F_{X_3}[K_3]) \right) + (\beta - 1) \left(-(1 - \theta)\lambda c_3 K_3 + \theta \left(-c_3 K_3 F_{X_3}[c_3 K_3/p] + p \int_0^{c_3 K_3/p} x_3 f_{X_3}(x_3) dx_3 \right) \right), \quad (15)$$

and the first and second derivatives are

$$\frac{dV_{LA}^{SF}(K_3)}{dK_3} = \theta(p(1 - F_{X_3}[K_3]) - (\beta - 1)c_3 F_{X_3}[c_3 K_3/p] - (\lambda\beta + (1 - \lambda\beta)\theta)c_3) \quad (16)$$

$$\frac{d^2V_{LA}^{SF}(K_3)}{dK_3^2} = \theta \left(-pf_{X_3}(K_3) - (\beta - 1)c_3 \cdot f_{X_3}\left(\frac{c_3 K_3}{p}\right)\left(\frac{c_3}{p}\right) \right) \leq 0. \quad (17)$$

The optimal flexible investment K_3^* is therefore given by

$$F_{X_3}[K_{3,LA}^{SF,*}] + (\beta - 1)\left(\frac{c_3}{p}\right)F_{X_3}\left[\frac{c_3 K_{3,LA}^{SF,*}}{p}\right] = 1 - \frac{(\lambda\beta + (1 - \lambda\beta)\theta)c_3}{\theta p}, \quad (18)$$

and the resulting objective value is

$$V_{LA}^{SF,*} = \theta p \left(\int_0^{K_{3,LA}^{SF,*}} x_3 f_{X_3}(x_3) dx_3 + (\beta - 1) \int_0^{(c_3/p)K_{3,LA}^{SF,*}} x_3 f_{X_3}(x_3) dx_3 \right). \quad (19)$$

We note that Equations (14) to (19) extend directly to the case where $N > 2$, with $N + 1$ replacing 3. We also note that Equation (18) collapses to the risk-neutral optimal investment (10) when $\beta = 1$.

Closed-form solutions for $K_{3,LA}^{SF,*}$ and $V_{LA}^{SF,*}$ will not exist in general. We have, however, been able to obtain closed-form solutions for the case of \tilde{X}_1 and \tilde{X}_2 having independent uniform distributions.

PROPOSITION 2. *Let \tilde{X}_1 and \tilde{X}_2 have independent $U(0, 1)$ distributions. If*

$$(p/2)(\theta - (\beta - 1)(c_3/p)^3) \leq (\lambda\beta + (1 - \lambda\beta)\theta)c_3,$$

then

$$K_{3,LA}^{SF,*} = \sqrt{\frac{2(1 - (\lambda\beta + (1 - \lambda\beta)\theta)c_3/(\theta_3 p))}{1 + (\beta - 1)(c_3/p)^3}}.$$

Otherwise,

$$K_{3,LA}^{SF,*} = \left[-1 + \left(1 - \frac{1}{2} \left((\beta - 1) \left(\frac{c_3}{p} \right)^3 - 1 \right) \cdot \left(-2 + \frac{(\lambda\beta + (1 - \lambda\beta)\theta)c_3}{\theta p} \right) \right)^{1/2} \right] \cdot \left[\frac{1}{2} \left((\beta - 1) \left(\frac{c_3}{p} \right)^3 - 1 \right) \right]^{-1}.$$

We now turn to the SD network. Let $w_n(x_n, y_n, K_n)$ be the realized profit generated by product n from an investment of K_n in dedicated resource n , and let $\tilde{W}_n(K_n)$ be the random profit. Then

$$w_n(x_n, y_n, K_n) = -(\lambda + (1 - \lambda)y_n)cK_n + p \min\{x_n, y_n K_n\}. \quad (20)$$

The random total profit for the SD network is $\tilde{W}^{\text{SD}}(K_1, K_2) = \tilde{W}_1(K_1) + \tilde{W}_2(K_2)$. The probability of a loss depends on the realization of \tilde{X}_1 , \tilde{X}_2 , \tilde{Y}_1 , and \tilde{Y}_2 . Because losses and gains are weighted differently, the loss-averse SD network cannot be decomposed into two single-product loss-averse problems as was possible in the risk-neutral case. The loss-averse objective function is

$$V_{\text{LA}}^{\text{SD}}(K_1, K_2) = E_{\tilde{X}, \tilde{Y}}[\tilde{W}^{\text{SD}}(K_1, K_2)] + (\beta - 1) \cdot E_{\tilde{X}, \tilde{Y}}[\tilde{W}^{\text{SD}}(K_1, K_2) | \tilde{W}^{\text{SD}}(K_1, K_2) < 0], \quad (21)$$

where

$$E_{\tilde{X}, \tilde{Y}}[\tilde{W}^{\text{SD}}(K_1, K_2)] = \sum_{n=1}^2 \left[-(\lambda + (1-\lambda)\theta)cK_n + \theta p(L_{X_n}(K_n) + K_n(1 - F_{X_n}(K_n))) \right], \quad (22)$$

$$L_{X_n}(z) = \int_0^z x_n f_{X_n}(x_n) dx_n \quad (23)$$

$$E_{\tilde{X}, \tilde{Y}}[\tilde{W}^{\text{SD}}(K_1, K_2) | \tilde{W}^{\text{SD}}(K_1, K_2) < 0] = (1-\theta)^2 G_{00}(K_1, K_2) + \theta(1-\theta)G_{10}(K_1, K_2) + (1-\theta)\theta G_{01}(K_1, K_2) + \theta^2 G_{11}(K_1, K_2). \quad (24)$$

We note that the $G_{y_1, y_2}(K_1, K_2)$ expressions can be found in Appendix D and that y_n denotes the realization (success or failure) of resource n in the $G_{y_1, y_2}(K_1, K_2)$ expressions. Closed-form solutions for $K_{n, \text{LA}}^{\text{SD},*}$ and $V_{\text{LA}}^{\text{SD},*}$ do not exist in general.

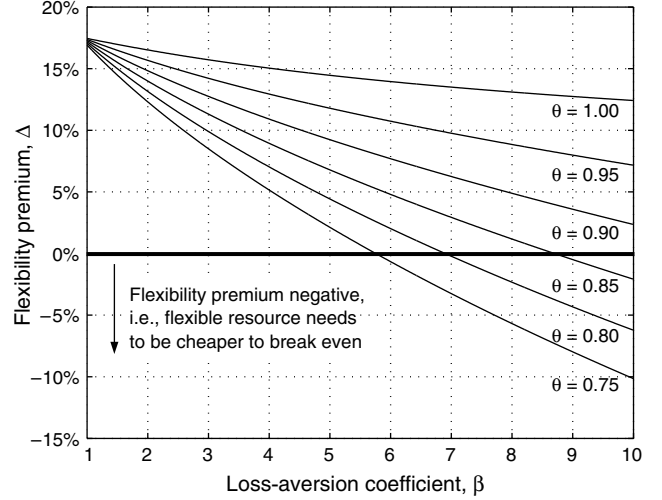
PROPOSITION 3. Let \tilde{X}_1 and \tilde{X}_2 have independent $U(0, 1)$ distributions. Then

$$K_{1, \text{LA}}^{\text{SD},*} = K_{2, \text{LA}}^{\text{SD},*} = -\gamma_A + \sqrt{\gamma_A^2 - \gamma_B}$$

where

$$p < 2c \Rightarrow \begin{cases} \gamma_A = \left[(\beta-1) \left(\left(4\frac{c}{p} - 4\left(\frac{c}{p}\right)^2 - 1 \right) \theta^2 - \frac{1}{2}\theta(1-\theta)^2(1+\lambda)^2 \left(\frac{c}{p}\right)^2 \right) - \theta \right] \\ \quad \cdot \left[(\beta-1)\theta^2 \left(\frac{3}{2} + 4\left(\frac{c}{p}\right)^3 - 5\frac{c}{p} \right) \right]^{-1} \\ \gamma_B = \left[\theta - \frac{c}{p}(\lambda + (1-\lambda)\theta + (\beta-1)(1-\theta)^2\lambda) \right] \\ \quad \cdot \left[(\beta-1)\theta^2 \left(\frac{3}{2} + 4\left(\frac{c}{p}\right)^3 - 5\frac{c}{p} \right) \right]^{-1} \end{cases}$$

Figure 4 Flexibility Premium for Loss-Averse Objective with $p/c = 2$ and $\lambda = 0.2$



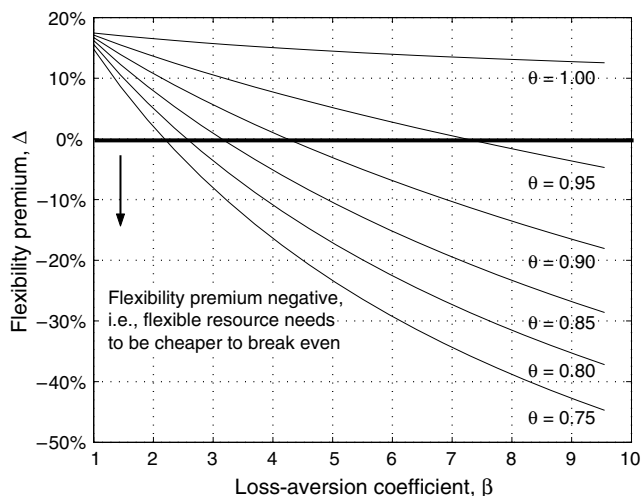
and

$$p \geq 2c \Rightarrow \begin{cases} \gamma_A = \frac{1}{2\theta} \left(\left(\frac{p}{c} \right)^3 \frac{1}{\beta-1} + \frac{p}{c} (1-\theta)(1+\lambda)^2 \right) \\ \gamma_B = \frac{1}{2(\beta-1)\theta} \left(\frac{p}{c} \right)^2 \\ \quad \cdot \left(\frac{\lambda}{\theta} + (1-\lambda) + (\beta-1)(1-\theta)^2 \frac{\lambda}{\theta} - \frac{p}{c} \right). \end{cases}$$

Figures 4 and 5 illustrate the dependence of the loss-averse flexibility premium Δ_{LA} on the reliability θ and the loss-aversion coefficient β .

Observe that Δ_{LA} can be negative (especially as the reliability decreases or loss aversion increases). This means that the SD network can be strictly preferred to the SF network even when the flexible resource costs the same or less than the dedicated resources; a result that cannot occur in the risk-neutral case (see Proposition 1). In the case of loss aversion, the downside resource-aggregation disadvantage of the flexible strategy is amplified by the higher weight placed on losses, with the result that SD can outperform SF. Comparing Figures 4 and 5, we see that moving from $\lambda = 0.2$ to $\lambda = 0.8$ makes SD preferable over a larger set of (θ, β) pairings, because the consequences of failure are larger when the marginal committed cost increases. We note that Δ_{LA} is not necessarily decreasing everywhere in β ; we observed Δ_{LA} to be increasing and then decreasing for instances with higher p/c values.

Figure 5 Flexibility Premium for Loss-Averse Objective with $p/c = 2$ and $\lambda = 0.8$



The insight that the flexible resource may have to be cheaper than the dedicated resources for the firm to prefer SF to SD does not hinge on the choice of a loss-averse objective function. Qualitatively similar results can be shown to hold under the CVaR objective; that is the flexibility premium can be negative. Specific propositions and results for the CVaR objective can be found in Appendix B.

3.3. Flexibility Premium in Perfectly Reliable Supply Chains

The reader may have noticed that the loss-averse flexibility premium Δ_{LA} is nonnegative everywhere for $\theta = 1$ in Figures 4 and 5. A similar phenomenon is also observed for the CVaR objective. Such numerical results suggest that the flexibility premium Δ is always nonnegative for $\theta = 1$. This suggestion is confirmed by the following proposition. Define U_1 as the set of utility functions that are nondecreasing in wealth, i.e., more is (weakly) better.

PROPOSITION 4. *Let the firm have an initial wealth of w_0 . Let $\tilde{\mathbf{X}}$ have any joint distribution. Let $\theta_1 = \theta_2 = \dots = \theta_N = \theta_{N+1} = 1$. Then (i) $\Delta \geq 0$ for all utility functions $u_1 \in U_1$, (ii) in particular $\Delta \geq 0$ for the loss-averse objective function and the CVaR objective function.*

Proposition 4 is a quite general result because it makes no assumptions on the demand distribution and only very mild assumptions on the utility function

(that it be nondecreasing in wealth). Clearly, U_1 contains all concave increasing utility functions; the common model for risk aversion. In fact U_1 contains all (locally and globally) risk-averse or risk-seeking utility functions as long as $u'(w) \geq 0$ everywhere.

It is commonly accepted intuition that a flexible strategy is preferable to a dedicated strategy if the investment costs are equal. This intuition manifests itself in the literature in the assumption that the unit cost for the flexible resource is higher than for dedicated resources. Using Propositions 1 and 4, we can establish when the common intuition is in fact valid.

REMARK 1. For the case of identical resources (marginal total costs, marginal committed costs, and reliabilities), the SF network is preferred to the SD network if either the resource investments are perfectly reliable (i.e., $\theta = 1$) or the firm is risk neutral. If neither condition holds, then the SD network can be strictly preferred to the SF network.

Thus, the common intuition is valid if either the risk neutrality or the perfect reliability assumption holds, but can be incorrect if neither condition holds.

4. Dual-Source Networks: DD and DF

We now consider the DD and DF networks. We choose to call these dual-source networks because the firm can invest in dual resources for any given product. We do not, however, force the firm to invest in all available resources, so a single-source strategy for one or more products may be optimal.

4.1. Two-Product DF Network

We label the resources as follows: resource 1 is dedicated to product 1, resource 2 is dedicated to product 2, and resource 3 is totally flexible. This DF network is an extension of Van Mieghem (1998), hereafter referred to as VM98, in that we allow for unreliable resource investments (with costs that are linear in both the target investment and the realized investment) and non-risk-neutral objective functions. The model collapses to that of VM98 if all reliabilities equal one and the objective value is $V_{RN}(\mathbf{K})$. VM98 restricts attention to $c_3 > \max\{c_1, c_2\}$, because otherwise the flexible resource clearly dominates at least one of the dedicated resources. We consider all $c_3 \geq 0$, because in our more general model investing in the dedicated resources may be optimal even if $c_3 \leq$

$\min\{c_1, c_2\}$. There are eight possible structures to the optimal solution (corresponding to different combinations of positive resource investments), and each of the eight structures can be optimal depending on the model parameters.

Let us consider the risk-neutral problem. For any feasible investment vector $\mathbf{K}=(K_1, K_2, K_3)$, the partial derivatives with respect to K_j are given by

$$\begin{aligned} \frac{\partial V_{RN}^{DF}(\mathbf{K})}{\partial K_1} &= -c_1(\lambda_1 + (1-\lambda_1)\theta_1) \\ &\quad + \theta_1(p_1((1-\theta_3)P(\Omega_1) + \theta_3P(\Omega_2)) \\ &\quad\quad + \theta_3p_2((1-\theta_2)P(\Omega_3) + \theta_2P(\Omega_4))) \\ \frac{\partial V_{RN}^{DF}(\mathbf{K})}{\partial K_2} &= -c_2(\lambda_2 + (1-\lambda_2)\theta_2) \\ &\quad + \theta_2p_2((1-\theta_3)P(\Omega_5) \\ &\quad\quad + \theta_3((1-\theta_1)(P(\Omega_6) + P(\Omega_7)) \\ &\quad\quad + \theta_1(P(\Omega_4) + P(\Omega_8) + P(\Omega_9)))) \\ \frac{\partial V_{RN}^{DF}(\mathbf{K})}{\partial K_3} &= -c_3(\lambda_3 + (1-\lambda_3)\theta_3) \\ &\quad + \theta_3p_1((1-\theta_1)P(\Omega_{10}) + \theta_1P(\Omega_2)) \\ &\quad + \theta_3p_2((1-\theta_1)(1-\theta_2)P(\Omega_{11}) + \theta_1(1-\theta_2) \\ &\quad\quad \cdot (P(\Omega_3) + P(\Omega_{12})) + (1-\theta_1)\theta_2P(\Omega_6) \\ &\quad\quad + \theta_1\theta_2(P(\Omega_4) + P(\Omega_9))), \end{aligned} \quad (25)$$

where each of the Ω_k , $k = 1, \dots, 12$, is a demand-space region, and the union of the Ω_k cover the demand space. We note that because of the Bernoulli failures, the problem does not lend itself to a strict partitioning (disjoint Ω_k) of the demand space as in Figure 1 of VM98. Expressions for the $P(\Omega_k)$ can be found in Appendix C. $V_{RN}^{DF}(\mathbf{K})$ can be shown to be jointly concave in (K_1, K_2, K_3) . Let $V_{RN}^{DF,*}$ denote the optimal objective value and $K_{j,RN}^{DF,*}$ the optimal investment level for resource $j = 1, 2, 3$. An interior solution ($K_{1,RN}^{DF,*} > 0, K_{2,RN}^{DF,*} > 0, K_{3,RN}^{DF,*} > 0$) is optimal if and only if the three derivatives in (25) all equal zero for some $(K_1 > 0, K_2 > 0, K_3 > 0)$. If such a solution exists, it is unique. If such a solution does not exist, then one or more of the $K_{j,RN}^{DF,*}$ must equal zero and so at least one of the products is single sourced. Closed-form solutions for $(K_{1,RN}^{DF,*}, K_{2,RN}^{DF,*}, K_{3,RN}^{DF,*})$ do not exist for the VM98 model and so will not exist for our model. We can, however, characterize the directional influence of

prices, marginal total costs, marginal committed-costs, reliabilities, and failure rebates on $V_{RN}^{DF,*}$ and $K_{j,RN}^{DF,*}$.

We first extend Propositions 3 and 4 of VM98 to characterize the directional influence of marginal total costs and prices on $V_{RN}^{DF,*}$ and $K_{j,RN}^{DF,*}$ when resource investments are unreliable.

PROPOSITION 5. For a risk-neutral firm, (i) $V_{RN}^{DF,*}$ is a nonincreasing convex function of the marginal total-cost vector \mathbf{c} and the marginal committed-cost vector $\boldsymbol{\lambda}$, and a nondecreasing convex function of the contribution-margin vector \mathbf{p} ; (ii) if the marginal demand density $f_{X_n}(x)$, $n = 1, 2$, is either log-concave or log-convex, then the directional sensitivity of the optimal investment vector $\mathbf{K}_{RN}^{DF,*}$ with respect to \mathbf{p} , \mathbf{c} , and $\boldsymbol{\lambda}$ is given by the following matrices (e.g., $\partial K_{1,RN}^{DF,*} / \partial c_3 \geq 0$)

$$\begin{aligned} \nabla_{\mathbf{p}} \mathbf{K}_{RN}^{DF,*} &= \begin{bmatrix} \geq 0 & \geq 0 & \geq 0 \\ \geq 0 & \geq 0 & \geq 0 \end{bmatrix}, \\ \nabla_{\mathbf{c}} \mathbf{K}_{RN}^{DF,*} &= \begin{bmatrix} \leq 0 & \leq 0 & \geq 0 \\ \leq 0 & \leq 0 & \geq 0 \\ \geq 0 & \geq 0 & \leq 0 \end{bmatrix}, \\ \nabla_{\boldsymbol{\lambda}} \mathbf{K}_{RN}^{DF,*} &= \begin{bmatrix} \leq 0 & \leq 0 & \geq 0 \\ \leq 0 & \leq 0 & \geq 0 \\ \geq 0 & \geq 0 & \leq 0 \end{bmatrix}. \end{aligned}$$

Actual expressions for the sensitivities are available on request, but are very involved and not particularly insightful. Proposition 5 proves that the perfect-reliability results of VM98 for the \mathbf{c} and \mathbf{p} vectors still hold for an unreliable network. As discussed in VM98, the presence of the flexible resource means that the optimal investment level for the resource dedicated to product n is influenced by the resource dedicated to product $3 - n$, $n = 1, 2$. We also see that the marginal total costs and committed costs have the same directional influence. VM98 does not address marginal committed costs because such costs are irrelevant in a perfectly reliable network. We note that the assumption of log-concavity or log-convexity for the marginal demand densities is a mild one that is met by the Uniform, Normal, Weibull, Gamma, Pareto, and Logistic distributions, as well as by many others.

PROPOSITION 6. For a risk-neutral firm, (i) $V_{RN}^{DF,*}$ is a nondecreasing convex function of the reliability vector θ ; (ii) if the marginal demand densities $f_{x_n}(x)$ are uniform, then the directional sensitivity of the optimal investment vector $\mathbf{K}_{RN}^{DF,*}$ with respect to θ is given by the following matrix:

$$\nabla_{\theta} \mathbf{K}_{RN}^{DF,*} = \begin{bmatrix} \geq 0 & \geq 0 & \leq 0 \\ \geq 0 & \geq 0 & \leq 0 \\ \leq 0 & \leq 0 & \geq 0 \end{bmatrix}.$$

As one would expect, the firm benefits from increasing resource reliabilities. We again see a cross-dependence for the dedicated resources because of the presence of the flexible resource. An increase in the reliability of a dedicated resource $j = 1, 2$ increases the investment in that resource j . This decreases the investment in the flexible resource, and as a consequence increases the investment in the other dedicated resource $3 - j$.

4.2. Two-Product DD Network

We now consider the DD network in which there are four dedicated resources ($j = 1, 2$ dedicated to product 1 and $j = 3, 4$ dedicated to product 2). There are 16 possible structures to the optimal solution (corresponding to different combinations of positive resource investments); each of the structures can be optimal depending on the model parameters.

This model is an extension of the single-period version of Model 1 in Anupindi and Akella (1993), hereafter referred to as AA93. Setting $\tilde{X}_2 = 0$ (i.e., single product), $\lambda_1 = \lambda_2 = 0$, and $V(\mathbf{K}) = V_{RN}(\mathbf{K})$ recovers the AA93 model. For any feasible investment vector $\mathbf{K} = (K_1, K_2, K_3, K_4)$, the partial derivatives (for the risk-neutral objective function) with respect to K_j are given by

$$\begin{aligned} \frac{\partial V_{RN}^{DD}(\mathbf{K})}{\partial K_j} &= -c_j(\lambda_j + (1 - \lambda_j)\theta_j) \\ &\quad + \theta_j(\theta_{3-j}(1 - F_{X_1}[K_1 + K_2]) \\ &\quad + (1 - \theta_{3-j})F_{X_1}[K_j]), \quad j = 1, 2 \end{aligned} \quad (26)$$

$$\begin{aligned} \frac{\partial V_{RN}^{DD}(\mathbf{K})}{\partial K_j} &= -c_j(\lambda_j + (1 - \lambda_j)\theta_j) \\ &\quad + \theta_j(\theta_{7-j}(1 - F_{X_2}[K_3 + K_4]) \\ &\quad + (1 - \theta_{7-j})F_{X_2}[K_j]), \quad j = 3, 4. \end{aligned} \quad (27)$$

$V_{RN}^{DD}(\mathbf{K})$ can be shown to be jointly concave in (K_1, K_2, K_3, K_4) . Let $V_{RN}^{DD,*}$ denote the optimal objec-

tive value and $K_{j,RN}^{DD,*}$ the optimal investment level for resource $j = 1, \dots, 4$. AA93 proved that there was no closed-form solution for the optimal investment levels in their model, and so there will not be a closed-form solution to our more general model. We can, however, still say something about the directional influence of the costs, reliabilities, and prices on these optimal values.

PROPOSITION 7. For a risk-neutral firm, (i) $V_{RN}^{DD,*}$ is a nonincreasing convex function of the marginal total-cost vector \mathbf{c} and the marginal committed-cost vector λ , and a nondecreasing convex function of the contribution-margin vector \mathbf{p} and the reliability vector θ ; (ii) the directional sensitivity of the optimal investment vector $\mathbf{K}_{RN}^{DD,*}$ with respect to \mathbf{p} , \mathbf{c} , λ , and θ is given by the following matrices

$$\begin{aligned} \nabla_{\mathbf{p}} \mathbf{K}_{RN}^{DD,*} &= \begin{bmatrix} \geq 0 & \geq 0 & = 0 & = 0 \\ \geq 0 & \geq 0 & = 0 & = 0 \end{bmatrix}, \\ \nabla_{\mathbf{c}} \mathbf{K}_{RN}^{DD,*} &= \begin{bmatrix} \leq 0 & \geq 0 & = 0 & = 0 \\ \geq 0 & \leq 0 & = 0 & = 0 \\ = 0 & = 0 & \leq 0 & \geq 0 \\ = 0 & = 0 & \geq 0 & \leq 0 \end{bmatrix}, \\ \nabla_{\lambda} \mathbf{K}_{RN}^{DD,*} &= \begin{bmatrix} \leq 0 & \geq 0 & = 0 & = 0 \\ \geq 0 & \leq 0 & = 0 & = 0 \\ = 0 & = 0 & \leq 0 & \geq 0 \\ = 0 & = 0 & \geq 0 & \leq 0 \end{bmatrix}, \\ \nabla_{\theta} \mathbf{K}_{RN}^{DF,*} &= \begin{bmatrix} \geq 0 & \leq 0 & = 0 & = 0 \\ \leq 0 & \geq 0 & = 0 & = 0 \\ = 0 & = 0 & \geq 0 & \leq 0 \\ = 0 & = 0 & \leq 0 & \geq 0 \end{bmatrix}. \end{aligned}$$

For the DF network, the investment level for dedicated resource for product $n = 1, 2$ was influenced by the costs and reliabilities of the dedicated resource for product $3 - n$, because of the coupling effect of the flexible resource. This proposition tells us that in the DD network, investment levels for a resource dedicated to product $n = 1, 2$ are influenced by the other dedicated resource for product n , but not by the dedicated resources for product $3 - n$.

4.3. Numerical Studies

Sections 4.1 and 4.2 established the directional influence of costs, reliabilities, and prices on the optimal profits and absolute investment levels within a given network structure, but they did not speak to either the influence of model parameters on the relative investment levels within a network structure or the relative attractiveness of the different network structures.

We address such questions through two numerical studies. For the case of discrete demand distributions (i.e., when probabilities are characterized by scenarios rather than densities), the firm's investment problem can be formulated as a scenario-based stochastic linear program. Formulations for the general model (for each of the three objective functions) are presented in Appendix E. We used this approach in the two numerical studies.

The first study was designed to address two types of questions. First, for a given network structure, we want to understand how the investment mix (i.e., percentage invested in each resource) is influenced by demand correlation, contribution-margin difference, reliability, and investment criterion. These were chosen because they are key drivers of the attractiveness of flexibility and dual sourcing. Second, we want to understand the relative value of the various network structures. For example, consider a firm that currently operates an SD network. How much benefit does it get by moving to a dual-sourcing network? Which dual-sourcing network is preferred, and under what circumstances? We fixed the number of products at $N = 2$ in this study. The second study was designed to investigate the relative performance of the four networks as the number of products increases.

We now describe the design and discuss the results for the first study. The demand distribution for each product was characterized by 200 demand scenarios. The demand scenarios were drawn randomly from a bivariate normal distribution. Because the demand variance was not a focus of this study, we fixed the demand mean and standard deviation to be 100 and 30, respectively, for both products. We varied the correlation coefficient (ρ) from -1 to 1 in increments of $1/3$, giving us a total of seven demand distributions that varied in their correlation coefficient. The actual correlations were -1.00 , -0.69 , -0.38 , -0.04 , 0.31 , 0.66 , and 1.00 . The contribution margin for product 2

was fixed at $p_2 = 10$. The relative contribution margin (p_1/p_2) was varied from 1.0 to 1.2 in increments of 0.05 , giving us a total of five contribution-margin ratios. In §§4.1 and 4.2, we analytically characterized the influence of a change in the reliability of a single resource, so we chose to investigate the influence of changes in the overall network reliability in the numerical studies. To that end, we assume that $\theta_j = \theta$ for all resources (resource failures are still independent) and vary θ from 0.2 to 1.0 in increments of 0.1 , giving us a total of nine supply-chain reliabilities. $\theta = 0.1$ was not used because it can be shown that all resources will have zero investment at this reliability level. We chose nine different objective functions: risk neutral, loss aversion ($\beta = 2, 3, 4, 5$), and CVaR ($\eta = 0.95, 0.9, 0.85, 0.8$). All other model parameters were held constant across problem instances because they were not the focus of the study. The marginal committed cost was fixed at $\lambda = 0.2$ for all resources. The marginal total costs were fixed at $c_j = 5$ for all dedicated resources and at $c_j = 6$ for all flexible resources. A full factorial set of tests was run for each network, so a total of $7 \times 5 \times 9 \times 9 = 2,835$ instances was solved for each of the four networks. The optimal solution and objective value for each problem instance was stored in a database that is available from the authors on request.

The investment-mix question is particularly relevant for the DF network. For the risk-neutral objective, Tables 1 to 3 present the percentage invested in the flexible resource,

$$\%K_{3,RN}^{DF,*} = \frac{K_{3,RN}^{DF,*}}{K_{1,RN}^{DF,*} + K_{2,RN}^{DF,*} + K_{3,RN}^{DF,*}},$$

where the numbers presented are medians for $\%K_{3,RN}^{DF,*}$ taken across the study instances. For example, there

Table 1 Influence of Correlation on Percent Investment in the Flexible Resource in the DF Network

ρ	$\%K_{3,RN}^{DF,*}$
-1.00	38
-0.69	38
-0.38	35
-0.04	33
0.31	31
0.66	30
1.00	29

Table 2 Influence of Relative Contribution Margin on Percent Investment in the Flexible Resource in the DF Network

p_1/p_2	$\%K_{3,RN}^{DF,*}$
1.00	30
1.05	32
1.10	33
1.15	34
1.20	36

were 315 risk-neutral instances for the DF network, so there were a total of 45 instances for each of the seven correlations. As can be seen, the median percentage invested in the flexible resource increased in the relative contribution margin and decreased in the demand correlation. Even for perfectly positively correlated demands, the median investment in the flexible resource was 29%. This number is driven by two factors. First, as shown by VM98, an asymmetric contribution margin can make the flexible resource attractive even if $\rho = 1$. Second, the flexible resource offers a diversification benefit in unreliable networks. For those instances with perfectly positively correlated demand and identical contribution margins (i.e., no flexibility benefit), the average flexible investment was 0% when $\theta = 1$ (i.e., perfectly reliable), but was 18% when $\theta = 0.8$. This demonstrates the importance of the diversification benefit that the flexible resource provides in the DF network when investments are unreliable. Reliability itself has a somewhat complex influence on $\%K_{3,RN}^{DF,*}$. For a given supply-chain reliability, the expected marginal investment cost $c_j(\lambda + (1 - \lambda)\theta)$ is 20% higher for the flexible resource than

Table 3 Influence of Reliability on Percent Investment in the Flexible Resource in the DF Network

θ	$\%K_{3,RN}^{DF,*}$
0.2	22
0.3	38
0.4	38
0.5	37
0.6	35
0.7	32
0.8	27
0.9	23
1.0	17

Table 4 Influence of Correlation on Percent of Instances in Which DF Is Better, Worse, or Equal to DD

ρ	$V_{RN}^{DF,*} > V_{RN}^{DD,*}$	$V_{RN}^{DF,*} < V_{RN}^{DD,*}$	$V_{RN}^{DF,*} = V_{RN}^{DD,*}$
-1.00	82.22	17.78	0.00
-0.69	77.78	22.22	0.00
-0.38	71.11	28.89	0.00
-0.04	64.44	35.56	0.00
0.31	44.44	55.56	0.00
0.66	17.78	82.22	0.00
1.00	0.00	88.89	11.11

for a dedicated resource (because $c_1 = c_2 = 5$, $c_3 = 6$). At low reliabilities, this higher cost outweighs any flexibility benefits provided by the flexible resource because the benefits only arise in the unlikely event that the resource investment succeeds. As reliability increases, there is initially a dramatic increase in $\%K_{3,RN}^{DF,*}$ because the flexibility and diversification benefits of the flexible resource become significant, but as the network reliability continues to improve there is a significant decrease in $\%K_{3,RN}^{DF,*}$ as the diversification benefit becomes less important (and nonexistent at $\theta = 1$). While the numbers in Tables 1 to 3 are presented for the risk-neutral objective, the same observations for the influence of correlation, relative margin, and reliabilities held for the loss-averse and CVaR objective functions.

We compared the optimal objective value across the networks for each problem instance. Given the study design, the DF network can never underperform the SD or SF networks, but can underperform the DD network. The DF network outperformed the SD network (i.e., $V_{RN}^{DF,*} > V_{RN}^{SD,*}$) in 93.65% of the 315 risk-neutral instances and the performance was equivalent in the other 6.35% of the 315 instances. The median expected-profit improvement was 14.63% over the 93.65% of cases for which $V_{RN}^{DF,*} > V_{RN}^{SD,*}$, indicating that a firm can gain significant advantage by moving from an SD

Table 5 Influence of Relative Margin on Percent of Instances in Which DF Is Better, Worse, or Equal to DD

p_1/p_2	$V_{RN}^{DF,*} > V_{RN}^{DD,*}$	$V_{RN}^{DF,*} < V_{RN}^{DD,*}$	$V_{RN}^{DF,*} = V_{RN}^{DD,*}$
1.00	38.10	60.32	1.59
1.05	44.44	53.97	1.59
1.10	50.79	47.62	1.59
1.15	57.14	41.27	1.59
1.20	65.08	33.33	1.59

Table 6 Influence of Network Reliability on Percent of Instances in Which DF Is Better, Worse, or Equal to DD

θ	$V_{RN}^{DF,*} > V_{RN}^{DD,*}$	$V_{RN}^{DF,*} < V_{RN}^{DD,*}$	$V_{RN}^{DF,*} = V_{RN}^{DD,*}$
0.2	0.00	100.00	0.00
0.3	8.57	91.43	0.00
0.4	42.86	57.14	0.00
0.5	57.14	42.86	0.00
0.6	62.86	37.14	0.00
0.7	68.57	31.43	0.00
0.8	65.71	34.29	0.00
0.9	68.57	31.43	0.00
1.0	85.71	0.00	14.29

network to a DF network. The DF network offers both flexibility and diversification benefits, so the question arises as to which type of benefit is really driving the superior performance. We investigated this question by creating a network with the same diversification benefit but no flexibility benefit. This was done by creating a special DD network in which the failures of resources 2 and 3 were perfectly positively correlated (i.e., if one failed, so did the other) and having their marginal total costs be identical to the flexible resource cost (i.e., $c_2 = c_3 = 6$). This network outperformed the SD network in 66.67% of the instances, and in those instances the median expected profit improvement was 3.48%. Comparing these results with those for the DF network, we see that, whereas the diversification benefit of the DF network is significant, the flexibility benefit is more significant.

Although the DF network provides some diversification, the (independent failure) DD network provides more diversification but no flexibility. For the risk-neutral objective, the DF network outperformed the DD network ($V_{RN}^{DF,*} > V_{RN}^{DD,*}$) in 51.11% of the instances, underperformed ($V_{RN}^{DF,*} < V_{RN}^{DD,*}$) in 47.30% of the instances, and performed equally in 1.59% of the instances. Such data might suggest that a firm

Table 7 Influence of Loss Aversion on Percent of Instances in Which DF Is Better, Worse, or Equal to DD

β	$V_{LA}^{DF,*} > V_{LA}^{DD,*}$	$V_{LA}^{DF,*} < V_{LA}^{DD,*}$	$V_{LA}^{DF,*} = V_{LA}^{DD,*}$
1	51.11	47.30	1.59
2	41.46	56.79	1.74
3	37.50	60.71	1.79
4	35.00	63.21	1.79
5	33.83	64.29	1.88

is somewhat indifferent between these two networks, but this is not the case. As Tables 4 to 7 demonstrate, the network preference is highly driven by the demand correlation, the relative contribution margin, the resource reliability, and the investment criterion. The DF network is much more attractive at lower demand correlations, higher relative margins, and higher reliabilities, whereas the DD network is much more attractive at higher demand correlations, lower relative margins, and lower reliabilities. The attractiveness of the DF network increases as either the demand correlation decreases or the relative contribution margin increases because the demand-pooling and contribution-margin benefits of flexibility are higher in such circumstances. The DF network is less attractive at low reliabilities because the extra level of diversification provided by the DD network is very beneficial if resources are very unreliable. The firm's investment criterion is also a key driver of network preference. As the firm becomes more loss averse, it prefers the DD network in a higher percentage of instances.

In the second study, we investigated the influence of the number of products on the relative performance of the four networks. This was done by fixing all other parameters and varying the number of products from two to five. Demand for each product was assumed to be independent (as the influence of correlation was established above) and was characterized by 200 demand scenarios randomly drawn from a normal distribution with mean and standard deviation of 100 and 30, respectively. Product contribution margins were assumed to be identical (as the influence of contribution-margin differences was established above) and equal to 10. Resource

Table 8 Influence of the Number of Products on the Relative Difference in Profit Between the DF and DD Networks

θ	$N = 2$	$N = 3$	$N = 4$	$N = 5$
0.3	-12.87	-9.03	-5.88	-3.68
0.4	-4.56	0.45	4.08	6.67
0.5	-1.43	3.90	7.46	10.12
0.6	-0.24	5.12	8.61	11.04
0.7	0.17	5.22	8.51	10.78
0.8	0.57	4.94	7.74	9.88
0.9	1.37	4.80	7.13	8.79
1.0	2.48	5.14	6.79	8.38

Table 9 Drivers of a Firm’s Preference for a Flexible or Dedicated Strategy

Element	Attribute	Influence on preference	Reason
Product portfolio	Demand correlations	Preference for DF increases as demands become more negatively correlated.	Negative correlation increases the demand-pooling benefit of the flexible resource in the DF network.
	Contribution margins	Preference for DF increases as the spread in contribution margins increases.	Wider margin range increases the contribution-margin option benefit (VM98) of the flexible resource in the DF network.
	Number of products	Preference for DF increases as the number of products increases.	The demand-pooling benefit of the flexible resource increases as the number of products increases.
Resources	Reliabilities	Preference for DF decreases as resource investments become less reliable.	Higher probability of resource failures increases the diversification benefits of the DD network.
Firm	Risk tolerance	Preference for DF decreases as firm becomes more concerned about downside risk.	In an unreliable network, the extra diversification provided by the DD network lowers the firm’s downside risk.

costs were assumed to be the same as in the above study. We chose a risk-neutral objective (as the influence of non-risk-neutral objectives was established above) and solved the investment problem for each of the four network structures for network reliabilities $\theta = 0.3, \dots, 1.0$ and number of products $N = 2, 3, 4, 5$. We then calculated the relative network performance as θ and N vary. The relative performance for the DF and DD networks ($100 \times (V_{RN}^{DF,*} - V_{RN}^{DD,*})/V_{RN}^{DD,*}$) can be found in Table 8. For any given value of θ , the relative performance of the DF network improves as the number of products increases. The reason for this is that the demand-pooling benefit of the flexibility resource increases with the number of products.

5. Conclusions

In this paper, we bridged the mix-flexibility and dual-sourcing literatures by studying four canonical supply-chain design strategies. Comparing the SD and SF networks, we identified the critical roles that risk tolerance and resource reliabilities play in the relative attractiveness of the two networks. We refined the prevailing intuition that an SF-type network is preferable to an SD-type network if a flexible resource is no more costly than a dedicated resource. In particular, we proved that the intuition is valid if either the resource investments are perfectly reliable or the firm is risk neutral, but the intuition can be wrong if neither condition holds. All things being equal (resource costs and reliabilities), a dedicated strategy can actually be strictly better than a flexible strategy. In fact, the dedicated strategy can be strictly better

even if dedicated resources are more expensive than a flexible resource.

We provided analytical results for the directional influence of prices, marginal costs (total and committed), and reliabilities on the optimal expected profit and resource levels for both the DD and DF networks. In contrast to the DD network, optimal dedicated-resource levels in the DF network are dependent on resources that are dedicated to other products, a result that suggests that flexible supply chains may not lend themselves to decentralized design as easily as do product-dedicated supply chains.

A comprehensive numerical study was undertaken to investigate how the attributes of three key supply-chain elements—namely, product portfolio, resources, and the firm—influence the desirability of a given design strategy. As one would expect, the desirability of a dual-sourcing network (DD versus SD or DF versus SF) increases as supply-chain reliability decreases. The story is more nuanced when comparing single-sourcing networks (SD versus SF) or dual-sourcing networks (DD versus DF). Table 9 summarizes the key results for such comparisons. We note that, whereas Table 9 is framed in terms of dual-sourcing networks, a similar story holds for single-sourcing networks.

We conclude by identifying two dimensions not considered in this paper. First, a firm’s enthusiasm for resource diversification might be dampened by scale economies in resource investments or by coordination costs in dealing with multiple suppliers for the same product. Second, supply-chain design may be a

decentralized rather than a centralized endeavor, and this raises the question of how to coordinate resource-investment decisions. We hope that future research will further refine the insights provided in this paper by addressing these and other considerations.

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Appendix A. Proofs

PROOF OF PROPOSITION 1. (i) $\Delta \geq 0 \Leftrightarrow V_{RN}^{SF,*} - V_{RN}^{SD,*} \geq 0$ at $c_{N+1} = c$. Using Equations (10), (11), (12), and (13), we then have

$$\Delta \geq 0 \Leftrightarrow \int_0^{F_{X_{N+1}}^{-1}[\alpha]} x_{N+1} f_{X_{N+1}}(x_{N+1}) dx_{N+1} - \sum_{n=1}^N \int_0^{K_n^*} x_n f_{X_n}(x_n) dx_n \geq 0, \quad (A-1)$$

where $\alpha = 1 - (\lambda + (1 - \lambda)\theta)c/\theta p$. Now, $\int_0^{F_X^{-1}[\alpha]} xf(x) dx = -\alpha ES_\alpha(X)$, where $ES_\alpha(X)$ is the α -expected shortfall for a continuous random variable X as defined in Acerbi and Tasche (2002, hereafter AT02). Note that $\alpha \in (0, 1)$. By substituting this into (A-1), rearranging the terms, and using the fact that $\tilde{X}_{N+1} = \sum_{n=1}^N \tilde{X}_n$, we then have

$$\Delta_{RN} \geq 0 \Leftrightarrow \alpha \left(\sum_{n=1}^N ES_\alpha(\tilde{X}_n) - ES_\alpha \left(\sum_{n=1}^N \tilde{X}_n \right) \right) \geq 0.$$

Now $ES_\alpha(X)$ is subadditive (see AT02), i.e.,

$$ES_\alpha \left(\sum_{n=1}^N \tilde{X}_n \right) \leq \sum_{n=1}^N ES_\alpha(\tilde{X}_n)$$

and so $\Delta_{RN} \geq 0$.

(ii) It is straightforward to show that the flexible-only resource level and the dedicated-only resource levels will be positive if and only if $\theta > \lambda c/(p - (1 - \lambda)c)$. Therefore,

$$\theta \leq \lambda c/(p - (1 - \lambda)c) \Rightarrow V_{RN}^{SF,*} - V_{RN}^{SD,*} = 0.$$

(iii) $\rho_{1n} = 1$ for $n = 2, \dots, N$. Therefore, for each for $n = 2, \dots, N$ we have $\tilde{X}_n = a_{1n}\tilde{X}_1 + b_{1n}$ for some constants a_{1n} and b_{1n} with $a_{1n} > 0$. Using the positive-homogeneity and translation-invariance properties of the expected shortfall measure, we have $ES_\alpha(a\tilde{X} + b) = aES_\alpha(\tilde{X}) - b$; see AT02, Proposition 3.1(iii) and (iv). We then have

$$\begin{aligned} \sum_{n=1}^N ES_\alpha(\tilde{X}_n) &= ES_\alpha(\tilde{X}_1) + \sum_{n=2}^N (a_{1n}ES_\alpha(\tilde{X}_1) - b_{1n}) \\ &= \left(1 + \sum_{n=2}^N a_{1n}\right) ES_\alpha(\tilde{X}_1) - \sum_{n=2}^N b_{1n}. \end{aligned}$$

Furthermore, $\sum_{n=1}^N \tilde{X}_n = (1 + \sum_{n=2}^N a_{1n})\tilde{X}_1 + \sum_{n=2}^N b_{1n}$ and so

$$ES_\alpha \left(\sum_{n=1}^N \tilde{X}_n \right) = \left(1 + \sum_{n=2}^N a_{1n}\right) ES_\alpha(\tilde{X}_1) - \sum_{n=2}^N b_{1n} = \sum_{n=1}^N ES_\alpha(\tilde{X}_n).$$

Therefore, $\Delta_{RN} = 0$. \square

PROOF OF PROPOSITION 2. If \tilde{X}_1 and \tilde{X}_2 are independent and identically distributed (i.i.d.) $U(0, 1)$, then $\tilde{X}_3 = \tilde{X}_1 + \tilde{X}_2$ has a triangular distribution over $(0, 2)$ and so $F_{X_3}[x] = x^2/2$ for $0 \leq x \leq 1$ and $F_{X_3}[x] = 2x - x^2/2 - 1$ for $1 < x \leq 2$. Proof follows from application of Equation (18). \square

PROOF OF PROPOSITION 3. Both resources are identical, so an optimal solution will have $K_{1,LA}^{SD,*} = K_{2,LA}^{SD,*}$. We therefore can restrict attention to investments of the form $K_1 = K_2 = K$, and K^* must be less than or equal to one because the demand is $U(0, 1)$. Using Equations (21)–(24) and the $G_{y_1, y_2}(K_1, K_2)$ equations in Appendix D, one can derive the following expressions for $V_{LA}^{SD}(K, K)$. If $p > 2c$, then

$$\begin{aligned} V_{LA}^{SD}(K, K) &= 2 \left((\theta p - c(\lambda + (1 - \lambda)\theta))K - \theta p \frac{K^2}{2} - (\beta - 1) \left((1 - \theta)^2 c \lambda K \right. \right. \\ &\quad \left. \left. + \frac{\theta(1 - \theta)(c(1 - \lambda)K)^2}{2p} + \frac{2\theta^2 c^3}{3p^3} K^3 \right) \right) \quad (A-2) \end{aligned}$$

and if $p \leq 2c$, then

$$\begin{aligned} V_{LA}^{SD}(K, K) &= 2 \left((\theta p - c(\lambda + (1 - \lambda)\theta))K - \theta p \frac{K^2}{2} - (\beta - 1) \left((1 - \theta)^2 c \lambda K \right. \right. \\ &\quad \left. \left. + \frac{\theta(1 - \theta)(c(1 - \lambda)K)^2}{2p} + \theta^2 \left(\left(\frac{5c}{6} - \frac{p}{4} - \frac{2c^3}{3p^3} \right) K^3 \right. \right. \right. \\ &\quad \left. \left. \left. - \left(2c - \frac{p}{2} - \frac{2c^2}{p} \right) K^2 \right) \right) \right). \quad (A-3) \end{aligned}$$

$V_{LA}^{SD}(K, K)$ is a concave (cubic) function in K (for both p/c regions) and so the first-order condition is sufficient for optimality. For both p/c regions the first-order condition is quadratic with one single nonnegative root. This root is the optimal investment level given in the proposition statement. \square

PROOF OF PROPOSITION 4. (i) Let $\mathbf{K}^{SD,*} = (K_1^*, \dots, K_N^*)$ be the optimal SD investment for some $u_1 \in U_1$. Consider the following feasible SF investment: $K_{N+1} = \sum_{n=1}^N K_n^*$. Let \tilde{W}^{SD} and \tilde{W}^{SF} be the profit random variables for the SD and SF network using these strategies, with distributions denoted by $F_{W^{SD}}[w]$ and $F_{W^{SF}}[w]$. For any demand realization $\mathbf{x} = (x_1, \dots, x_N)$, the SD and SF networks will have the following terminal wealths:

$$w^{SD}(\mathbf{K}^{SD,*}) = w_0 + p \sum_{n=1}^N \min\{x_n, K_n^*\} - c \sum_{n=1}^N K_n^* \quad (A-4)$$

$$w^{SF} \left(\sum_{n=1}^N K_n^* \right) = w_0 + p \min \left\{ \sum_{n=1}^N x_n, \sum_{n=1}^N K_n^* \right\} - c \sum_{n=1}^N K_n^*, \quad (A-5)$$

so

$$\begin{aligned} & w^{\text{SF}}\left(\sum_{n=1}^N K_n^*\right) - w^{\text{SD}}(\mathbf{K}^{\text{SD},*}) \\ &= p\left(\min\left\{\sum_{n=1}^N x_n, \sum_{n=1}^N K_n^*\right\} - \sum_{n=1}^N \min\{x_n, K_n^*\}\right) \geq 0. \quad (\text{A-6}) \end{aligned}$$

Therefore, $F_{W^{\text{SF}}}[w] \leq F_{W^{\text{SD}}}[w]$, so \tilde{W}^{SF} first-order stochastically dominates \tilde{W}^{SD} . Therefore,

$$E[u_1^{\text{SD}}(\mathbf{K}^{\text{SD},*})] \leq E\left[u_1^{\text{SF}}\left(\sum_{n=1}^N K_n^*\right)\right]$$

for all utility functions $u_1 \in U_1$ (see Equation (4) in Levy 1992) and so

$$E[u_1^{\text{SD}}(\mathbf{K}^{\text{SD},*})] \leq E[u_1^{\text{SF}}(K_{N+1}^*)].$$

Therefore $\Delta \geq 0$.

(ii) The loss-averse objective function $V_{\text{LA}}(\mathbf{K})$ is identical to an expected utility objective function where the utility function is piecewise-linear increasing (with the breakpoint at w_0). Such a utility function is in the U_1 set, so the loss-averse result follows directly from Proposition (4). \tilde{W}^{SF} first-order stochastically dominates \tilde{W}^{SD} , so \tilde{W}^{SF} second-order stochastically dominates \tilde{W}^{SD} . Recalling that CVaR is the average value of the profit falling below the η -percentile level, we can use Theorem 3 in Levy (1992) to state that

$$V_{\text{CVaR}}^{\text{SD}}(K_1^*, \dots, K_N^*) \leq V_{\text{CVaR}}^{\text{SF}}\left(\sum_{n=1}^N K_n^*\right).$$

Therefore,

$$V_{\text{CVaR}}^{\text{SD}}(K_1^*, \dots, K_N^*) \leq V_{\text{CVaR}}^{\text{SF}}(K_{N+1}^*),$$

so $\Delta \geq 0$ for the CVaR objective function. \square

PROOF OF PROPOSITION 5. Proof of this proposition follows the same structure as that of VM98 Propositions 3 and 4. $V_{\text{RN}}^{\text{DF}}(\mathbf{K})$ is nonincreasing convex in $\mathbf{c} = (c_1, c_2, c_3)$ on the convex set \mathbb{R}_3^+ for each $\mathbf{K} \in \mathbb{R}_3^+$. $V_{\text{RN}}^{\text{DF},*}(\mathbf{c})$ is therefore nonincreasing convex in \mathbf{c} as maximization preserves convexity. A similar argument holds for $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$. $V_{\text{RN}}^{\text{DF}}(\mathbf{K})$ is nondecreasing convex in $\mathbf{p} = (p_1, p_2)$ on the convex set $p \geq p_2 \geq 0$ for each $\mathbf{K} \in \mathbb{R}_3^+$. $V_{\text{RN}}^{\text{DF},*}(\mathbf{p})$ is therefore nondecreasing convex in \mathbf{p} as maximization preserves convexity. The Hessian matrix for $V_{\text{RN}}^{\text{DF}}(\mathbf{K})$ is

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix},$$

where

$$\begin{aligned} h_{11} &= -\theta_1(p_1(\theta_3 I_4(K_1, K_2, K_3) + (1 - \theta_3)(I_4(K_1, K_2, 0) \\ &\quad + I_6(K_1, K_2, 0))) + p_2(\theta_2 \theta_3 (I_2(K_1, K_2, K_3) \\ &\quad + I_5(K_1, K_2, K_3)) + (1 - \theta_2) \theta_3 (I_2(K_1, 0, K_3) \\ &\quad + I_5(K_1, 0, K_3) - I_4(K_1, K_2, K_3))) \\ &\quad + (p_1 - p_2) \theta_3 I_6(K_1, K_2, K_3)) \end{aligned}$$

$$h_{12} = h_{21} = -\theta_1 \theta_2 \theta_3 I_2(K_1, K_2, K_3)$$

$$\begin{aligned} h_{13} = h_{31} &= -\theta_1 \theta_3 (p_1 I_4(K_1, K_2, K_3) + p_2 (\theta_2 I_2(K_1, K_2, K_3) \\ &\quad + (1 - \theta_2)(I_2(K_1, 0, K_3) - I_4(K_1, K_2, K_3))) \\ &\quad + (p_1 - p_2) I_6(K_1, K_2, K_3)) \end{aligned}$$

$$\begin{aligned} h_{22} &= -\theta_2 p_2 (\theta_1 \theta_3 (I_1(K_1, K_2, K_3) + I_2(K_1, K_2, K_3) + I_3(K_1, K_2, K_3)) \\ &\quad + (1 - \theta_1) \theta_3 (I_2(0, K_2, K_3) + I_3(0, K_2, K_3)) \\ &\quad + (1 - \theta_3)(I_1(K_1, K_2, 0) + I_3(K_1, K_2, 0))) \end{aligned}$$

$$\begin{aligned} h_{23} = h_{32} &= -\theta_2 \theta_3 p_2 (\theta_1 (I_1(K_1, K_2, K_3) + I_2(K_1, K_2, K_3)) \\ &\quad + (1 - \theta_1)(I_2(0, K_2, K_3))) \end{aligned}$$

$$\begin{aligned} h_{33} &= -\theta_3 (p_1 (\theta_1 I_4(K_1, K_2, K_3) + (1 - \theta_1)(I_4(0, K_2, K_3))) \\ &\quad + p_2 (\theta_1 \theta_2 (I_1(K_1, K_2, K_3) + I_2(K_1, K_2, K_3)) \\ &\quad + \theta_1 (1 - \theta_2)(I_1(K_1, 0, K_3) + I_2(K_1, 0, K_3) \\ &\quad - I_4(K_1, K_2, K_3)) + (1 - \theta_1) \theta_2 I_2(0, K_2, K_3) \\ &\quad + (1 - \theta_1)(1 - \theta_2)(I_2(0, 0, K_3) - I_2(0, K_2, K_3))) \\ &\quad + (p_1 - p_2)(\theta_1 I_6(K_1, K_2, K_3) + (1 - \theta_1) I_6(0, K_2, K_3))) \end{aligned}$$

where the line integrals $I_k(K_1, K_2, K_3)$ $k = 1, \dots, 6$ are given in Appendix C and are similar to the I_k expressions in the proof of Proposition 1 in VM98. Because we have to generalize for the case of investment failures, we write the function in terms of $I_k(K_1, K_2, K_3)$ rather than simply I_k .

Proof of the gradients of K_j^* with respect to \mathbf{c} , $\boldsymbol{\lambda}$, and \mathbf{p} follows from use of the Implicit Function Theorem (IFT) for each of the eight possible optimal solution structures. Application of the IFT requires detailed algebra that is available from the authors on request. We note that the interior solution structure ($K_1^* > 0, K_2^* > 0, K_3^* > 0$) is by far the most cumbersome, and the only one in which we needed the sufficient condition of log-convexity or log-concavity for the proof. \square

PROOF OF PROPOSITION 6. Proof of this proposition follows the same structure as that of Proposition 5. $V_{\text{RN}}^{\text{DF}}(\mathbf{K})$ is nondecreasing convex in $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ on the convex set $0 \leq \theta_j \leq 1, j = 1, \dots, 3$, for each $\mathbf{K} \in \mathbb{R}_3^+$. $V_{\text{RN}}^{\text{DF},*}(\mathbf{K})$ is therefore nondecreasing convex in $\boldsymbol{\theta}$ as maximization preserves convexity. Proof of the gradients of K_j^* with respect to $\boldsymbol{\theta}$ follows

from use of the IFT for each of the eight possible optimal solution structures. Application of the IFT requires detailed algebra that is available from the authors on request. We note that the interior solution structure ($K_1^* > 0, K_2^* > 0, K_3^* > 0$) is by far the most cumbersome and the only one in which we used the sufficient condition of i.i.d. uniform demands for the proof. \square

PROOF OF PROPOSITION 7. $V_{RN}^{DD}(\mathbf{K})$ is nonincreasing convex in \mathbf{c} (and in $\boldsymbol{\lambda}$) on the convex set \mathbb{R}_3^+ for each $\mathbf{K} \in \mathbb{R}_3^+$. $V_{RN}^{DD,*}(\mathbf{K})$ is therefore nonincreasing convex in \mathbf{c} (and in $\boldsymbol{\lambda}$) as maximization preserves convexity. $V_{RN}^{DD}(\mathbf{K})$ is nondecreasing convex in \mathbf{p} (and in $\boldsymbol{\theta}$) on the convex set $p \geq p_2 \geq 0$ ($0 \leq \theta_j \leq 1, j = 1, \dots, 3$) for each $\mathbf{K} \in \mathbb{R}_3^+$. $V_{RN}^{DF,*}(\mathbf{K})$ is therefore nondecreasing convex in \mathbf{p} (and in $\boldsymbol{\theta}$) as maximization preserves convexity. $V_{RN}^{DD}(\mathbf{K})$ is separable in (K_1, K_2) and (K_3, K_4) , i.e., the problem decomposes into two single-product problems. The resources dedicated to product i are thus independent of resources dedicated to product $3 - i$, $i = 1, 2$. Proof of the gradients of K_j^* follows from use of the IFT for each of the four possible optimal solution structures for the single-product problem; the detailed algebra is available from the authors on request. \square

Appendix B. The Flexibility Premium for the CVaR Measure

Chen et al. (2003) prove that $K^* = F_X^{-1}[\eta(p - c)/(p - v)]$ is the optimal investment for a perfectly reliable single-product newsvendor problem under a CVaR η objective, where v is the salvage value (assumed to be zero in our work). We extend this result to allow for Bernoulli investment failures. Recall that $\tilde{X}_{N+1} = \tilde{X}_1 + \tilde{X}_2 + \dots + \tilde{X}_N$ and c_{N+1} is the marginal total cost for the flexible resource.

PROPOSITION 8. *For the SF network, the optimal investment level for the CVaR η objective is*

$$K_{3,CVaR\eta}^* = F_{X_{N+1}}^{-1} \left[\frac{\eta}{\theta} \left(1 - \frac{c_3}{p} \right) + \frac{1 - \theta}{\theta} \left(\frac{c_{N+1}}{p} (1 - \lambda) - 1 \right) \right]. \quad (\text{B-1})$$

PROOF. Available from the authors on request. \square

We note that at $\theta = 1$ (B-1) gives the perfect-reliability investment of Chen et al. (2003) for the case of zero salvage value, and it also gives the risk-neutral optimal investment (10) at $\eta = 1$.

Using Equation (7), the SD investment problem can be formulated as

$$V_{CVaR\eta}^{SD,*} = \max_{K_1 \geq 0, \dots, K_N \geq 0, v} \left\{ v + \frac{1}{\eta} E_{\tilde{X}, \tilde{Y}} [\min\{\tilde{W}_{SD}(K_1, \dots, K_N) - v, 0\}] \right\}. \quad (\text{B-2})$$

As with loss aversion, the SD investment problem cannot be decomposed into N single-product problems. For the two-product case, one can show that

$$\begin{aligned} & E_{\tilde{X}, \tilde{Y}} [\min\{\tilde{W}_{SD}(K_1, K_2) - v, 0\}] \\ &= (1 - \theta_1)(1 - \theta_2)G_{00}(K_1, K_2, v) + \theta_1(1 - \theta_2)G_{10}(K_1, K_2, v) \\ &+ (1 - \theta_1)\theta_2G_{01}(K_1, K_2, v) + \theta_1\theta_2G_{11}(K_1, K_2, v), \quad (\text{B-3}) \end{aligned}$$

where the $G_{y_1, y_2}(K_1, K_2, v)$ expressions can be found in Appendix D. Numerical results (available from the authors on request) prove that the flexibility premium Δ_{CVaR} can be negative, so SD can be strictly preferable to SF even when the flexible resource costs the same or less than the dedicated resources, a result that echoes the loss-averse case. In the case of CVaR, the downside resource-disaggregation benefit of the dedicated strategy is amplified by the fact that the left tail of the profit distribution is always factored in, whereas the right tail is not, with the result that SD can outperform SF.

Appendix C. Expressions for $P(\Omega_k)$ and Line Integrals $I_k(K_1, K_2, K_3)$

$$P(\Omega_1) = \int_{K_1}^{\infty} \int_0^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_2) = \int_{K_1+K_3}^{\infty} \int_0^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_3) = \int_{K_1}^{K_1+K_3} \int_{K_1+K_3-x_1}^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_4) = \int_{K_1}^{K_1+K_3} \int_{K_1+K_2+K_3-x_1}^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_5) = \int_0^{\infty} \int_{K_2}^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_6) = \int_0^{K_3} \int_{K_2+K_3-x_1}^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_7) = \int_{K_3}^{\infty} \int_{K_2}^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_8) = \int_{K_1+K_3}^{\infty} \int_{K_2}^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_9) = \int_0^{K_1} \int_{K_2+K_3}^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_{10}) = \int_{K_3}^{\infty} \int_0^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_{11}) = \int_0^{K_3} \int_{K_3-x_1}^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$P(\Omega_{12}) = \int_0^{K_1} \int_{K_3}^{\infty} f_X(x_1, x_2) dx_2 dx_1$$

$$I_1(K_1, K_2, K_3) = \int_0^{K_1} f_X(x_1, K_2 + K_3) dx_1$$

$$I_2(K_1, K_2, K_3) = \int_{K_1}^{K_1+K_3} f_X(x_1, K_1 + K_2 + K_3 - x_1) dx_1$$

$$I_3(K_1, K_2, K_3) = \int_{K_1+K_3}^{\infty} f_X(x_1, K_2) dx_1$$

$$I_4(K_1, K_2, K_3) = \int_0^{K_2} f_X(K_1 + K_3, x_2) dx_2$$

$$I_5(K_1, K_2, K_3) = \int_{K_2+K_3}^{\infty} f_X(K_1, x_2) dx_2$$

$$I_6(K_1, K_2, K_3) = \int_{K_2}^{\infty} f_X(K_1 + K_3, x_2) dx_2$$

Appendix D. $G_{y_1 y_2}(K_1, K_2)$ and $G_{y_1 y_2}(K_1, K_2, v)$ Expressions

We present the expressions for the general case where the marginal committed costs (λ) and marginal total costs (c) can differ for resources 1 and 2. The $G_{y_1 y_2}(K_1, K_2)$ expressions found in Equation (24) in the loss-averse section are given by $G_{y_1 y_2}(K_1, K_2) = G_{y_1 y_2}(K_1, K_2, v = 0)$, where $G_{y_1 y_2}(K_1, K_2, v)$ are given below. We present the more general $G_{y_1 y_2}(K_1, K_2, v)$ expressions because they are used in the CVaR analysis.

$$G_{00}(K_1, K_2, v) = \begin{cases} 0 & \text{for } v < -(c_1 \lambda_1 K_1 + c_2 \lambda_2 K_2) \\ -(c_1 \lambda_1 K_1 + c_2 \lambda_2 K_2 + v) & \text{for } v \geq -(c_1 \lambda_1 K_1 + c_2 \lambda_2 K_2) \end{cases}$$

$$G_{10}(K_1, K_2, v) = \begin{cases} pL_{x_2} \left(\frac{c_1 \lambda_1 K_1 + c_2 K_2 + v}{p} \right) - (c_2 K_2 + c_1 \lambda_1 K_1 + v) \cdot F_{X_2} \left[\frac{c_1 \lambda_1 K_1 + c_2 K_2 + v}{p} \right] & \text{for } v \leq (p - c_2)K_2 - c_1 \lambda_1 K_1 \\ p(L_{x_2}(K_2) + K_2(1 - F_{X_2}[x_2])) - (c_2 K_2 + c_1 \lambda_1 K_1 + v) & \text{for } v > (p - c_2)K_2 + c_1 \lambda_1 K_1 \end{cases}$$

$$G_{01}(K_1, K_2) = \begin{cases} pL_{x_2} \left(\frac{c_1 \lambda_1 K_1 + c_2 K_2 + v}{p} \right) - (c_2 K_2 + c_1 \lambda_1 K_1 + v) F_{X_2} \left[\frac{c_1 \lambda_1 K_1 + c_2 K_2 + v}{p} \right] & \text{for } v \leq (p - c_2)K_2 - c_1 \lambda_1 K_1 \\ p(L_{x_2}(K_2) + K_2(1 - F_{X_2}[x_2])) - (c_2 K_2 + c_1 \lambda_1 K_1 + v) & \text{for } v > (p - c_2)K_2 + c_1 \lambda_1 K_1 \end{cases}$$

$$v \leq \min\{(p - c_1)K_1 - c_2 K_2, (p - c_2)K_2 - c_1 K_1\} \Rightarrow$$

$$G_{11}(K_1, K_2) = p \left(N_{X_1, X_2} \left(\frac{c_1 K_1 + c_2 K_2 + v}{p} \right) + O_{X_1, X_2} \left(\frac{c_1 K_1 + c_2 K_2 + v}{p} \right) - (c_1 K_1 + c_2 K_2 + v) M_{X_1, X_2} \left(\frac{c_1 K_1 + c_2 K_2 + v}{p} \right) \right)$$

$$(p - c_2)K_2 - c_1 K_1 < v \leq (p - c_1)K_1 - c_2 K_2 \Rightarrow$$

$$G_{11}(K_1, K_2) = pL_{x_1} \left(\frac{c_1 K_1 - (p - c_2)K_2 + v}{p} \right) + p(L_{x_2}(K_2) + K_2(1 - F_{X_2}[K_2])) \cdot F_{X_1} \left[\frac{c_1 K_1 - (p - c_2)K_2 + v}{p} \right] - (c_1 K_1 + c_2 K_2 + v) F_{X_1} \left[\frac{c_1 K_1 - (p - c_2)K_2 + v}{p} \right]$$

$$+ p \left(N_{X_1, X_2} \left(\frac{c_1 K_1 - (p - c_2)K_2 + v}{p}, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) + O_{X_1, X_2} \left(\frac{c_1 K_1 - (p - c_2)K_2 + v}{p}, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) \right) - (c_1 K_1 + c_2 K_2 + v) \cdot M_{X_1, X_2} \left(\frac{c_1 K_1 - (p - c_2)K_2 + v}{p}, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right)$$

$$(p - c_1)K_1 - c_2 K_2 < v \leq (p - c_2)K_2 - c_1 K_1 \Rightarrow$$

$$G_{11}(K_1, K_2) = p \left(N_{X_1, X_2} \left(\frac{c_1 K_1 + c_2 K_2 + v}{p} \right) - N_{X_1, X_2} \left(K_1, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) + O_{X_1, X_2} \left(\frac{c_1 K_1 + c_2 K_2 + v}{p} \right) - O_{X_1, X_2} \left(K_1, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) \right) - (c_1 K_1 + c_2 K_2 + v) \cdot \left(M_{X_1, X_2} \left(\frac{c_1 K_1 + c_2 K_2 + v}{p} \right) - M_{X_1, X_2} \left(K_1, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) \right)$$

$$+ \left((pK_1 - c_1 K_1 - c_2 K_2 - v) F_{X_2} \left[\frac{c_2 K_2 - (p - c_1)K_1 + v}{p} \right] + pL_{x_2} \left(\frac{c_2 K_2 - (p - c_1)K_1 + v}{p} \right) \right) (1 - F_{X_1}[K_1])$$

$$v > \max\{(p - c_1)K_1 - c_2 K_2, (p - c_2)K_2 - c_1 K_1\} \Rightarrow$$

$$G_{11}(K_1, K_2) = pL_{x_1} \left(\frac{c_1 K_1 - (p - c_2)K_2 + v}{p} \right) + p(L_{x_2}(K_2) + K_2(1 - F_{X_2}[K_2])) F_{X_1} \left[\frac{c_1 K_1 - (p - c_2)K_2 + v}{p} \right] - (c_1 K_1 + c_2 K_2 + v) F_{X_1} \left[\frac{c_1 K_1 - (p - c_2)K_2 + v}{p} \right]$$

$$+ p \left(N_{X_1, X_2} \left(\frac{c_1 K_1 - (p - c_2)K_2 + v}{p}, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) - N_{X_1, X_2} \left(K_1, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) \right) + p \left(O_{X_1, X_2} \left(\frac{c_1 K_1 - (p - c_2)K_2 + v}{p}, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) - O_{X_1, X_2} \left(K_1, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) \right) - (c_1 K_1 + c_2 K_2)$$

$$\cdot \left(M_{X_1, X_2} \left(\frac{c_1 K_1 - (p - c_2)K_2 + v}{p}, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) - M_{X_1, X_2} \left(K_1, \frac{c_1 K_1 + c_2 K_2 + v}{p} \right) \right)$$

$$+ \left((pK_1 - c_1 K_1 - c_2 K_2 - v) F_{X_2} \left[\frac{c_2 K_2 - (p - c_1)K_1 + v}{p} \right] + pL_{x_2} \left(\frac{c_2 K_2 - (p - c_1)K_1 + v}{p} \right) \right) (1 - F_{X_1}[K_1])$$

where

$$M_{X_1, X_2}(w_1, w_2) = \int_{w_1}^{w_2} F_{X_2}[w_2 - x_1] f_{X_1}(x_1) dx_1,$$

$$N_{X_1, X_2}(w_1, w_2) = \int_{w_1}^{w_2} x_1 F_{X_2}[w_2 - x_1] f_{X_1}(x_1) dx_1,$$

$$O_{X_1, X_2}(w_1, w_2) = \int_{w_1}^{w_2} L_{X_2}[w_2 - x_1] f_{X_1}(x_1) dx_1,$$

$$M_{X_1, X_2}(w) = M_{X_1, X_2}(0, w),$$

$$N_{X_1, X_2}(w) = N_{X_1, X_2}(0, w)$$

and

$$O_{X_1, X_2}(w) = O_{X_1, X_2}(0, w).$$

Appendix E. Linear Program Formulations for Investment Problem

There are a total of $M = 2^J$ possible yield scenarios. A yield scenario $m = 1, \dots, M$ is defined by a vector (y_1^m, \dots, y_J^m) where $y_j^m \in \{0, 1\}$, with $y_j^m = 0$ denoting failure. The probability of scenario m is denoted by ρ^m . Let there be I different demand scenarios, each defined by a demand vector (x_1^i, \dots, x_N^i) , with probability ρ^i , $i = 1, \dots, I$. If the demand scenarios are randomly generated from some joint distribution \tilde{X} , then $\rho^i = 1/I$ for $i = 1, \dots, I$. By assumption, yields are independent of demands and so we have a total of MI yield-demand scenarios, each specified by an mi pair with an associated probability of $\rho^{mi} = \rho^m \rho^i$.

Risk-neutral linear program (LP) formulation:

$$\max \sum_{i=1}^I \sum_{m=1}^M \rho^{mi} w^{mi}$$

subject to

$$w^{mi} = \sum_{n=1}^N p_n s_n^{mi} - \sum_{j=1}^J c_j (\lambda_j + (1 - \lambda_j) y_j^m) K_j, \\ m = 1, \dots, M, i = 1, \dots, I \quad (\text{E-1})$$

$$s_n^{mi} \leq x_n^i, \quad n = 1, \dots, N, m = 1, \dots, M, i = 1, \dots, I \quad (\text{E-2})$$

$$s_n^{mi} \leq \sum_{j=1}^J t_{nj} q_{nj}^{mi}, \quad n = 1, \dots, N, m = 1, \dots, M, i = 1, \dots, I \quad (\text{E-3})$$

$$\sum_{n=1}^N q_{nj}^{mi} \leq y_j^m K_j, \quad j = 1, \dots, J, m = 1, \dots, M, i = 1, \dots, I \quad (\text{E-4})$$

$$K_j, s_n^{mi}, q_{nj}^{mi} \geq 0, \quad n = 1, \dots, N, j = 1, \dots, J, \\ m = 1, \dots, M, i = 1, \dots, I. \quad (\text{E-5})$$

Loss-averse LP formulation:

$$\max \sum_{i=1}^I \sum_{m=1}^M \rho^{mi} (w^{mi,+} - \beta w^{mi,-})$$

subject to

$$w^{mi,+} - w^{mi,-} = w^{mi}, \quad m = 1, \dots, M, i = 1, \dots, I \quad (\text{E-6})$$

$$w^{mi,+}, w^{mi,-} \geq 0, \quad m = 1, \dots, M, i = 1, \dots, I$$

$$\text{and (E-1), (E-2), (E-3), (E-4), (E-5).} \quad (\text{E-7})$$

CVaR LP formulation:

$$\max \left\{ v + \frac{1}{\eta} \sum_{i=1}^I \sum_{m=1}^M \rho^{mi} z^{mi} \right\}$$

subject to

$$z^{mi} \leq w^{mi} - v^{mi}, \quad m = 1, \dots, M, i = 1, \dots, I \quad (\text{E-8})$$

$$z^{mi} \leq 0, \quad m = 1, \dots, M, i = 1, \dots, I$$

$$\text{and (E-1), (E-2), (E-3), (E-4), (E-5).} \quad (\text{E-9})$$

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