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# Pricing and Operational Recourse in Coproduction Systems

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Coproduction systems, in which multiple products are produced simultaneously in a single production run, are prevalent in many industries. Such systems typically produce a random quantity of vertically differentiated products. This product hierarchy enables the firm to fill demand for a lower-quality product by converting a higher-quality product. In addition to the challenges presented by random yields and multiple products, coproduction systems often serve multiple customer classes that differ in their product valuations. Furthermore, the sizes of these classes are uncertain. Employing a utility-maximizing customer model, we investigate the production, pricing, downconversion, and allocation decisions in a two-class, stochastic-demand, stochastic-yield coproduction system. For the single-class case, we establish that downconversion will not occur if prices are set optimally. In contrast, we show that downconversion can be optimal in the two-class case, even if prices are set optimally. We consider the benefit of postponing certain operational decisions, e.g., the pricing or allocation-rule decisions, until after uncertainties are resolved. We use the term recourse to denote actions taken after uncertainties have been resolved. We find that recourse pricing benefits the firm much more than either downconversion or recourse allocation do, implying that recourse demand management is more valuable than recourse supply management. Special cases of our model include the single-class and two-class random-yield newsvendor models.

*Key words:* flexibility; random yield; utility-maximizing customers

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## 1. Introduction

A key feature of many biochemical-, chemical-, and material-processing operations is that multiple products are produced simultaneously in a single production run. Such systems are sometimes referred to as coproduction systems (Bitran and Leong 1992, Bitran and Gilbert 1994). In the energy industry, the term coproduction is sometimes used to refer to the practice in which consumers of electricity also produce energy that is routed to the grid. That usage differs from the intended meaning of coproduction in this paper, that being the simultaneous production of multiple products.

Semiconductor manufacturing is one example of a coproduction system. The fabrication process produces random quantities of devices of varying speeds. (A device's speed is the result of random events that occur during fabrication.) Devices are then tested and binned according to their processor speed, with higher bins containing higher-speed devices. This results in a nested product structure that gives the firm some flexibility in meeting demand: "depending

on the demand in the marketplace, fast devices can be configured to run more slowly, but slow devices can not be enticed to run faster" (Kempf 2004, p. 4564). Filling demand for a low-speed device with a reconfigured high-speed device is sometimes referred to as downbinning. We note that downbinning "a product whereby its speed is lowered by blowing fuses in the chip" (Johnson 2005, p. 16) is a fast operation, on the order of days, whereas fabrication can take up to three months (Johnson 2005, Wu et al. 2006). As such, the fabrication quantity decision is made before yield and demand uncertainties are resolved, but the downbinning decision occurs after uncertainties are resolved (Johnson 2005).

Downbinning is valuable to the firm when it has an excess of high-speed devices, but a shortage of low-speed ones. However, why would a firm incur the downbinning (reconfiguration) cost? It could, instead, provide the low-speed customer a high-speed device at the low-speed price (and the customer would willingly accept this direct substitution). Such direct substitution, however, cannibalizes the high-speed

demand as some customers may be willing to purchase the high-speed device at the high-speed price if the low-speed device is sold out. There are also strategic reasons for firms to reconfigure high-speed devices before using them to fill low-speed demands. For example, reconfiguration prevents opportunistic reselling of high-speed devices that were obtained at low-speed prices. Reconfiguration is a common practice in the semiconductor industry. See Kempf (2004) and Bean et al. (2005) for references to the practice at Intel.

Semiconductor devices vary greatly in their life cycles depending on their end use. “The life cycle for custom ICs on cellular phones is quite different from that for the motor controller ICs for hard drives” (Wu et al. 2006, p. 235), with the life cycle of devices used in fashion products, e.g., phones, being very short—on the order of months. For such devices, the life cycle can be short enough relative to the production lead time that firms cannot replenish their inventory during the life cycle. For other devices, the life cycle is long enough to allow periodic replenishment.

A similar story plays out in many other coproduction systems. A single production run results in random quantities of vertically differentiated products. By vertical differentiation, we mean that the products differ along a key performance dimension, e.g., speed, for which all customers agree that more is better (at the same price). The nested product structure allows the firm to fill demand for a lower-quality product by either converting a high-quality product or by directly substituting the higher-quality product. We use the term downconversion to refer to the practice of converting a higher-quality product to a lower-quality one, and we use the term downgrading to refer to the practice of direct substitution. In this paper, we focus on situations in which the firm uses downconversion instead of downgrading, either because of the strategic concern of opportunistic reselling or because the downconversion cost is low relative to the cannibalization loss associated with downgrading. We do, however, investigate a model with downgrading in the online companion to this paper.

The management of coproduction systems is complicated by the fact that firms typically sell to heterogeneous customers, that is, customers differ in their valuation of quality. Customers with the same valuation can be thought of as being members of a particular customer class. The number of customers in each class may be uncertain. Customers not receiving their preferred product may choose to purchase some other product that is available.

Existing literature on coproduction systems has concerned itself with the production quantity and substitution decisions under the following assumptions: prices are exogenous, the preferred product of

each customer class is exogenous, each customer class has a different preferred product, unfilled demand does not spill over to other products, and there is no cost to downconversion. We note that zero-cost downconversion and downgrading are equivalent in these models as unfilled demand does not spill over. Bitran and Dasu (1992), Bitran and Leong (1992), and Bitran and Gilbert (1994) all propose various heuristic solutions to the multiple-period, deterministic-demand, N-product problem. Gerchak et al. (1996) investigate the structural properties of the optimal solution to the single-period, deterministic-demand, two-product problem. Hsu and Bassok (1999) and Rao et al. (2004) develop algorithms and heuristics for optimizing the single-period, stochastic-demand, N-product problem.

In this paper, we consider the pricing decision in coproduction systems and allow for spillover of unmet demand. Furthermore, we allow for costly downconversion. We note that when prices are endogenous and/or when customer spillover is considered, multiple customer classes might wish to purchase the same grade of product, a situation that cannot occur in the papers cited above. This highlights a further complication in managing coproduction systems: firms must decide how to allocate their inventories (after downconversion) among competing customer classes. Thus, there are four key decisions in managing a coproduction system: product pricing, production quantity, downconversion quantities, and the allocation rule.

With both supply and demand uncertainties present in coproduction systems, mismatches in realized supply and demand are to be expected. After uncertainties have been resolved, the firm might be able to take some action to better match realized supply and demand. We use the term operational recourse to describe any action taken after uncertainty is resolved. Downconversion is one such operational recourse; it allows the firm to adjust the proportion of the various products after uncertainties are resolved. The firm may have other operational recourses available to it. Rather than pricing in advance of uncertainty resolution, the firm may choose to postpone pricing until after uncertainties are resolved. Likewise, it may choose to postpone its decision regarding the allocation rule. The postponement of decisions until uncertainties are resolved is sometimes referred to as operational hedging (Van Mieghem 2003, Ding et al. 2007). Because ours is a risk-neutral setting, we prefer to use the term operational recourse as it more precisely reflects the salient feature of these uncertainty-management approaches. The implementation of recourse actions is not the only option for improving the performance of coproduction systems. A firm might instead (or in addition)

consider operational-improvement actions such as yield-uncertainty reduction, product-quality improvement, and production-cost reduction. Therefore, in measuring the value of any given recourse action (for example moving from advanced pricing to recourse pricing), we compare the value obtained to that obtained by operational-improvement actions.

In this paper, we embed a demand model of utility-maximizing customers in a single-period, two-product, two-customer-class, stochastic-demand coproduction system. Our utility-maximizing demand model ensures that a customer's purchasing behavior is completely consistent with her valuation of quality and the product prices. We establish that downconversion is not optimal in the single-class case if prices are set optimally (but downconversion can be optimal if prices are not set optimally). In contrast, downconversion can be optimal in the two-class case, even if prices are set optimally. We show that a priority-based allocation policy is optimal for recourse allocation regardless of the timing of the pricing decision. Of the three potential recourse actions, we find that recourse pricing delivers significantly more benefit than downconversion or recourse allocation. In addition, the value of either downconversion or recourse allocation is significantly reduced if recourse pricing is implemented. Relative to operational improvement actions, numeric results show that implementing recourse pricing delivers approximately the same benefit as completely eliminating the yield uncertainty, and a benefit equivalent to a 9% reduction in production cost, a 7% increase in expected yield of the high-quality product, or a 38% increase in the perceived quality of the low-grade product.

The rest of this paper is organized as follows. Section 2 surveys the existing related literature. Section 3 introduces the model. We consider a single-customer-class version of the model in §4, and then the two-customer-class version in §5. Conclusions and opportunities for future research are discussed in §6. An analysis of a number of special cases of our model and an investigation of some alternative models can be found in Online Appendices A–F. All appendices, including proofs (Appendix G), are provided in the e-companion.<sup>1</sup> An unabridged version of this paper containing some additional results is available upon request.

## 2. Literature

As the existing coproduction literature has been discussed in the introduction, we now focus on three other streams of literature related to this paper: the

joint quantity-and-price setting problem, investment in flexible resources, and utility-maximizing demand models in operations.

While the joint quantity-and-price setting problem has been studied quite extensively in the literature, most of the papers assume perfectly reliable supply and a single product. Petruzzi and Dada (1999), Van Mieghem and Dada (1999), and Dana and Petruzzi (2001) all consider the quantity-and-price setting problem in a single-product, perfect-supply newsvendor setting. Of these papers, only Van Mieghem and Dada (1999) consider operational recourse actions. Van Mieghem and Dada (1999) show that production postponement is of little value if the firm also engages in price postponement. Thomas (1974), Federgruen and Heching (1999), Polatoglu and Sahin (2000), Zhao and Wang (2002), Chen and Simchi-Levi (2004), Monahan et al. (2004), and Xu and Hopp (2004) all study single-product, perfect-supply problems, but their focus is on the relative benefit of dynamic pricing over static pricing in a multiperiod setting.

The joint quantity-and-price setting problem with multiple products has received much less attention. Bish and Wang (2004) and Chod and Rudi (2005) consider the quantity-and-price setting problem in a perfect-supply, two-product newsvendor model where the quantity is set before demand uncertainty is resolved, but price is set after the demand uncertainty is resolved. Advanced pricing is not considered. Bish and Wang (2004) consider dedicated and flexible resources, and they show that it can be optimal for the firm to invest in the flexible resource even with perfectly correlated demands. Chod and Rudi (2005) focus on a single flexible resource, and they prove that the optimal resource level is increasing in both demand variability and correlation. Both of these papers use price-dependent aggregate demand models, and implicitly assume horizontal rather than vertical product differentiation. In Bish and Wang (2004), demand for one product is not influenced by the price of the other product. In essence, they implicitly assume that there are two customer classes, and each class is willing to buy one product and one product only. Chod and Rudi (2005) do allow for cross-price demand dependencies. Aggregate demand models, as used in these two papers, would not be reasonable in the case of vertical differentiation because such models would not a priori rule out unreasonable situations such as there being demand for a lower-quality product even if its price is higher than that of a higher-quality product. Our utility-maximizing demand model rules out any such inconsistencies.

To the best of our knowledge, Li and Zheng (2006) is the only paper to consider the joint quantity-and-price setting problem in the presence of supply uncertainty. They investigate a single-product,

<sup>1</sup> An electronic companion to this paper is available as part of the online version that can be found at <http://mansci.journal.informs.org/>.

periodic-review model, where inventory replenishment and pricing decisions are set at the beginning of each period. They do not consider multiple products nor operational recourses. We note that Kazaz (2004) considers a single-product production-planning problem under yield uncertainty, with emergency procurement as a recourse action, but price is an exogenous function of realized yield.

Mix flexibility, whereby a resource (inventory or capacity) has the ability to produce multiple products, has been the subject of much attention in the operations literature. A resource may be totally flexible (i.e., it can fill demand for any product), partially flexible (i.e., it can fill demand for a subset of products), or dedicated (i.e., it can only fill demand for a particular product). Many papers (Fine and Freund 1990; Gupta et al. 1992; Li and Tirupati 1994, 1995, 1997; Van Mieghem 1998, 2004; Tomlin and Wang 2005) focus on investments in dedicated and totally flexible resources. Jordan and Graves (1995) and Graves and Tomlin (2003) investigate how the structure of partial flexibility in supply chains influences performance. These papers all assume exogenous prices. Bish and Wang (2004) and Chod and Rudi (2005) consider the product-pricing decision in the context of totally flexible resources.

Coproduction systems with vertical differentiation result in a special case of partial flexibility, whereby products are downwardly flexible. Netessine et al. (2002) investigate the resource-investment problem in a downwardly flexible system in the context of services such as rental cars. Unlike in coproduction systems, resource investments are not subject to yield uncertainty, and each resource requires a separate investment. Downgrading rather than downconversion is assumed, although these specific terms are not used. Prices are exogenous in Netessine et al. (2002). We note that all of the flexibility papers cited are single-period models.

The operations management literature has typically assumed that demand is independent of a firm's operating decisions. Linking demand to operating decisions can be achieved with aggregate demand functions, as done by Bish and Wang (2004) and Chod and Rudi (2005), or by explicitly modeling individual customers as utility-maximizing entities. This latter approach has been used by van Ryzin and Mahajan (1999) and Mahajan and van Ryzin (2001b) to determine the optimal assortment and stocking levels of products in a single-period problem with exogenous prices. The first paper assumes customers not receiving their first-choice products are lost. The second paper allows consumers to spill over to other products. Mahajan and van Ryzin (2001a) adopt a consumer-choice model to study inventory

competition in a multiple-firm, single-period problem with exogenous prices. Talluri and van Ryzin (2004) use a consumer-choice model in an airline revenue-management setting to study the assortment (fares to make available) problem in a finite-horizon, fixed-capacity problem. Dana and Petruzzi (2001) is the only paper of which we are aware that uses a utility-maximizing customer model to investigate a joint quantity-and-pricing problem. They study a single-product, perfect-supply newsvendor problem in which customers decide between purchasing a firm's product (at price  $p$ ) or some outside option. Ex ante, all customers are identical, that is, they have the same deterministic valuation  $V$  for the firm's product and the same distribution  $G(u)$  [density  $g(u)$ ] for the utility of their outside option. As such, there is a single class of customers.  $G(u)$  is assumed to satisfy the condition  $R'(\cdot) < R(\cdot)^2$ , where  $R(\cdot)$  is the reverse hazard rate function, i.e.,  $R(\cdot) = g(\cdot)/G(\cdot)$ . Ex post, customers differ depending on the realized utility of their outside option. A customer will purchase the product (if available) if  $V - p \geq u$ , but will walk away otherwise. Our paper builds upon this utility-maximizing model to allow for two products and two customer classes.

We now summarize the contribution of this paper. In addition to extending the coproduction literature by considering pricing, downconversion, and allocation, our paper extends the other three streams of literature as follows. To the best of our knowledge, it is the first paper to consider the joint quantity-and-price setting problem for multiple products in the presence of supply uncertainty, the first to consider the joint quantity-and-price setting problem in the class of downwardly flexible systems, and the first to use a utility-maximizing customer model for the joint quantity-and-price setting problem in either uncertain-supply or multiproduct settings.

### 3. The Model

We consider a monopolist newsvendor that sells two products (H and L) in a market with two classes of utility-maximizing customers. The two products differ in their quality level, with H being a higher quality than L. We use the term quality in the broad sense of any performance dimension for which all customers agree that more is better, i.e., we assume a vertical differentiation between the products. The firm sells product H at a price of  $p_H$  and product L at a price of  $p_L$ . We assume that the firm cannot price discriminate between customer classes, that is, it has to offer the same price to both classes. We will discuss the timing of the firm's pricing decision after we have first described the firm's production system and the utility-maximizing model that specifies customer-purchasing behaviors.

The firm operates a coproduction system in which the two products are produced simultaneously in a single production run. A production run of size  $Q$  produces  $q_H = yQ$  units of product H and  $q_L = (1 - y)Q$  units of product L, where  $y$  is the realization of a yield random variable  $\tilde{Y}$ . The yield random variable has a distribution function  $F_Y(\cdot)$  with support between 0 and 1. Production thus results in a random split between the two products. Our yield random variable is sometimes referred to as the split factor in the semiconductor industry (e.g., Wang et al. 2004). The total production cost is linear in the quantity launched, with the marginal production cost given by  $c_p$ . We note that all results carry through if the production cost is convex in  $Q$ . After production, the firm has the opportunity to downconvert product H to product L at a marginal cost of  $c_D$  per unit converted. One can think of product H as being a flexible or a customizable product. After downconversion, the firm then has an inventory of  $q_H - q_D$  units of product H and  $q_L + q_D$  units of product L, where  $q_D \leq q_H$  is the quantity of H converted to L.

The market consists of two customer classes (1 and 2). The market potential (or size) of each class is uncertain, with  $F_X(\cdot)$  denoting the joint distribution function for the market potentials. Customers within a class are infinitesimal and are homogeneous with respect to their valuations of the two products. Customer class  $i = 1, 2$  has a valuation of  $a_{iH}$  for product H and  $a_{iL}$  for product L. We assume that  $a_{iH} \geq a_{iL}$  for  $i = 1, 2$ , reflecting the fact that product H is of higher quality. Without loss of generality, we index the classes such that  $a_{1H} \geq a_{2H}$ . The utility that a class  $i$  customer derives from purchasing product H (or L) is then  $a_{iH} - p_H$  (or  $a_{iL} - p_L$ ). Each customer also has an outside option that provides her with a utility of  $u \in \mathcal{R}^+$ . Ex ante, all customers within a class are identical, but ex post they differ in the utility of their outside option and, hence, differ in their willingness to pay. Let the distribution of outside-option utilities for class  $i$  customers be  $G_i(\cdot)$ . We assume that  $G_1(\cdot)$  and  $G_2(\cdot)$  have independent, continuous distributions, but allow them to be nonidentically distributed. In addition, we assume that  $R'_i(\cdot) < R_i(\cdot)^2$  for  $i = 1, 2$ , where  $R_i(\cdot) = g_i(\cdot)/G_i(\cdot)$ . We note that  $R'_i(\cdot) < 0$  is equivalent to  $G_i(\cdot)$  being log concave, and so all log-concave distribution functions satisfy  $R'_i(\cdot) < R_i(\cdot)^2$ . Log-concave distribution functions include the Uniform, Normal, Weibull, Gamma, and many other common distributions. Truncations of log-concave distributions are also log concave (Bagnoli and Bergstrom 2005).

Customers observe their outside utility before making their purchasing decision. In an infinite-supply environment, a customer would choose the option (purchase H, purchase L, or take the outside option)

that maximizes her utility. In a limited-supply environment, a customer may not get her first-choice option due to inventory limitations. We assume the outside option is always available and, therefore, supply limitations are driven by the final quantities of H and L that result from the firm's production realization and downconversion decision. If a customer's first choice is unavailable, she then chooses among her remaining two options to again maximize her utility. If her second choice is unavailable, the customer is then lost to the firm.

The firm has four decisions to make in managing this system: the production quantity  $Q$  to launch, the postproduction downconversion quantity  $q_D$ , the pricing vector  $\mathbf{p} = (p_H, p_L)$  to charge, and the allocation or rationing rule to use if both customer classes demand the same product. We discuss each decision in turn.

- *Production quantity ( $Q$ ):* This decision must be made before market uncertainty, i.e., the size of each customer class, is resolved.

- *Downconversion quantity ( $q_D$ ):* This decision is made after both production-yield and market-size uncertainties have been resolved, and can be used to better balance the supply and demand for the two products. In Online Appendix F, we consider a model in which there is some residual market uncertainty when downconversion occurs. We note that the firm cannot simply downgrade product H, that is, sell H to a customer at the price of product L; it must first convert H to L. Downgrading is considered in Online Appendix E. We assume that all customer purchasing decisions are made instantaneously, and so the firm must complete any downconversion before customer purchasing begins. (An alternative model that relaxes this assumption is considered in the unabridged version of the paper.)

- *Price vector  $\mathbf{p} = (p_H, p_L)$ :* We consider two alternatives for the timing of the pricing decision. In the advanced-pricing case, prices are set before either yield uncertainty or market-size uncertainty has been resolved. In the recourse-pricing case, prices are set after both yield and market-size uncertainties have been resolved. This provides the firm with another lever to balance supply and demand. We note that the optimal prices must satisfy  $p_L^* < \max\{a_{1L}, a_{2L}\}$  and  $p_H^* < a_{1H}$  because otherwise no customer will purchase L or H. We, therefore, restrict attention to  $p_L < \max\{a_{1L}, a_{2L}\}$  and  $p_H < a_{1H}$  in all that follows. For a given price vector  $\mathbf{p} = (p_H, p_L)$ , customer class  $i = 1, 2$  prefers H to L if  $a_{iH} - p_H \geq a_{iL} - p_L$ , but prefers L to H if  $a_{iH} - p_H < a_{iL} - p_L$ . We use the convention that a customer who is indifferent between H and L will first try to purchase H. Therefore, we say that class  $i$  prefers H to L iff  $a_{iH} - p_H \geq a_{iL} - p_L$ . The distribution for the utility of class  $i$  customers' outside option is represented by  $G_i(\cdot)$ , and so, for a given price vector

$\mathbf{p} = (p_H, p_L)$ , the fraction of customers who prefer H to their outside option is  $G_i(a_{iH} - p_H)$ . For product L, the fraction is  $G_i(a_{iL} - p_L)$ . If  $a_{iH} - p_H \geq a_{iL} - p_L$ , then  $G(a_{iL} - p_L)/G(a_{iH} - p_H)$  is the fraction of those class  $i$  customers whose first choice is H who also prefer L to their outside option, i.e., it is the fraction of class  $i$  customers who spill over from H to L. If  $a_{iL} - p_L > a_{iH} - p_H$ , then the fraction of class  $i$  customers who spill over from L to H is  $G(a_{iH} - p_H)/G(a_{iL} - p_L)$ .

- *Allocation:* In the event of the firm having insufficient inventory of a particular product to meet demand of both classes, the firm must allocate or ration its inventory. As with pricing, we consider advance allocation and recourse allocation. Under advance allocation, the firm determines its allocation policy before uncertainties are resolved. Under recourse allocation, the firm determines its allocation policy after both production-yield and market-size uncertainties have been resolved.

We note that this model generalizes a number of newsvendor models that, to the best of our knowledge, have not been studied previously. Please see Figure 1. The lower right quadrant (CPP2) is the general model whereas special cases are contained in the other quadrants. We analyze CPP1 in §4 and CPP2 in §5. We analyze the random-yield special cases, RYP1 and RYP2, in Online Appendices A and B, respectively. Deterministic-supply versions of all these cases are obtained by setting the yield random variable  $y$  equal to some  $0 \leq \bar{y} \leq 1$  with probability 1.

In closing, it is worth commenting on two aspects of our model. The first aspect that merits discussion is our assumption that market uncertainty is fully resolved before the firm makes its recourse decisions, i.e., the downconversion quantity and the prices in the recourse-pricing case. Although the assumption of full uncertainty resolution has precedence in the operations literature (e.g., Van Mieghem and Dada 1999, Desai et al. 2007), and can be a reasonable approximation of reality, there may be situations in which

there is residual market uncertainty when the firm has to make its recourse decisions. For further discussion and an analysis of a model with residual uncertainty, we refer the reader to Online Appendix F. The second aspect that merits discussion is our customer model. In effect, we assume that each customer class contains a population of infinitesimally small entities even though customers of “coproduction firms” are businesses rather than end consumers. This is a reasonable approximation if the firm sells to a large number of small customers but is less reasonable if the firm sells to a very small number of large customers. Although this approximation has the advantage of tractability, some caution should be exercised before directly applying our customer model to a situation in which a firm sells primarily to a few key customers.

#### 4. A Single-Class Coproduction Model with Pricing

In this section, we consider the single-class coproduction model (labeled CPP1), and therefore remove the class subscript  $i = 1, 2$  from all parameters. Because there is only one class, no allocation decision is needed. The relevant decisions facing the firm are the price vector  $\mathbf{p} = (p_H, p_L)$ , the production quantity  $Q$ , and the downconversion quantity  $q_D$ . We first characterize the optimal downconversion quantity for a given price vector  $\mathbf{p}$ , realized yield  $y$ , and realized market potential  $x$ . We then proceed to consider the quantity-and-price setting problem. This section concludes with a numerical investigation of the impact of recourse pricing and operational improvements.

##### 4.1. The Optimal Downconversion Quantity

The firm chooses the downconversion quantity  $q_D$  after yield and market uncertainties have been resolved. The realized quantity of product H is  $q_H = yQ$ , the realized quantity of product L is  $q_L = (1 - y)Q$ , and the realized market potential is  $x$ . The firm chooses the downconversion quantity  $0 \leq q_D \leq q_H$  to maximize  $r(q_D) - c_D q_D$ , where the revenue function

$$r(q_D) = p_H \min\{xG(a_H - p_H), q_H - q_D\} + p_L \min\left\{[xG(a_H - p_H) - (q_H - q_D)]^+ \cdot \left(\frac{G(a_L - p_L)}{G(a_H - p_H)}\right), q_L + q_D\right\} \quad (1)$$

if  $a_H - p_H \geq a_L - p_L$  (i.e., customers prefer H to L), and

$$r(q_D) = p_L \min\{xG(a_L - p_L), q_L + q_D\} + p_H \min\left\{[xG(a_L - p_L) - (q_L + q_D)]^+ \cdot \left(\frac{G(a_H - p_H)}{G(a_L - p_L)}\right), q_H - q_D\right\} \quad (2)$$

Figure 1 Four Models

	Single product	Two products
Single class	<p>RYP1 (single-class, random-yield model with pricing)</p> <p><math>a_{1L} = 0; a_{1H} &gt; 0</math> <math>a_{2L} = 0; a_{2H} = 0</math></p>	<p>CPP1 (single-class, coproduction model with pricing)</p> <p><math>a_{1L} \geq 0; a_{1H} &gt; 0</math> <math>a_{2L} = 0; a_{2H} = 0</math></p>
Two classes	<p>RYP2 (two-class, random-yield model with pricing)</p> <p><math>a_{1L} = 0; a_{1H} &gt; 0</math> <math>a_{2L} = 0; a_{2H} \geq 0</math></p>	<p>CPP2 (two-class, coproduction model with pricing)</p> <p><math>a_{1L} \geq 0; a_{1H} &gt; 0</math> <math>a_{2L} \geq 0; a_{2H} \geq 0</math></p>



otherwise. The first term in each case is the revenue from sales of the first-choice product and the second term is the revenue from sales of the second-choice product. Define  $\alpha = G(a_H - p_H)/G(a_L - p_L)$ .

**THEOREM 1.** *The optimum downconversion quantity  $q_D^*$  is specified by one of the following three cases: (i)  $a_H - p_H \geq a_L - p_L \Rightarrow q_D^* = 0$ , (ii)  $a_H - p_H < a_L - p_L$  and  $c_D \geq p_L - \alpha p_H \Rightarrow q_D^* = 0$ , (iii)  $a_H - p_H < a_L - p_L$  and  $c_D < p_L - \alpha p_H \Rightarrow q_D^* = \min\{z, [q_H - z\alpha]^+ / (1 - \alpha)\}$ , where  $z = [xG(a_L - p_L) - q_L]^+$  is the amount of customers whose first choice, L in this case, is not satisfied.*

We note that downconversion never occurs if customers prefer H to L, because either all demand is satisfied from the inventory of H, in which case there is no reason to convert, or else all of product H inventory is sold, in which case there is nothing left to convert. Even if customers prefer L to H, then downconversion can occur only in the case where demand for L,  $xG(a_L - p_L)$ , exceeds  $q_L$  and the downconversion cost is sufficiently low.

Figure 2 illustrates the firm's revenue as a function of the downconversion quantity  $q_D$  for this case. As the downconversion quantity increases, the quantity of L increases, and so there is less unsatisfied first-choice demand. At the same time, the quantity of H decreases, and so there is less product available to meet spillover demand from unsatisfied customers. The revenue from sales of L is nondecreasing in  $q_D$ , whereas the revenue from sales of H is nonincreasing. In region 1, i.e.,  $q_H - q_D > (z - q_D)\alpha$ , the downconversion quantity is low enough such that all spillover demand is met and there is leftover inventory of H. The marginal increase in revenue from sales of L more than offsets the marginal loss in revenue from sales of H, and so revenue is increasing in  $q_D$ . In region 2, the downconversion quantity is high enough such that all remaining H is used and there is unsatisfied

spillover demand. The marginal increase in revenue from sales of L is less than the marginal loss in revenue from sales of H because the firm could have sold the converted unit at the higher price of  $p_H$  rather than selling it at  $p_L$ . The firm's revenue is maximized at the boundary of regions 1 and 2, which occurs at  $q_D = [q_H - z\alpha]^+ / (1 - \alpha)$ . We note that region 1 does not exist if  $q_H - z\alpha \leq 0$ . In this case, there is insufficient H to meet spillover demand even before downconversion occurs, and so  $q_D^* = 0$ .

**4.2. The Quantity-and-Pricing Problem**

In this section we investigate the firm's quantity-and-pricing problem. In the recourse-pricing case, the firm sets prices after yield and market uncertainties are resolved. In the advanced-pricing case, the firm sets prices before any uncertainties are resolved. We refer the reader to Online Appendix C for a treatment of advanced pricing, and focus attention here on the recourse-pricing case.

After yield and market uncertainties are resolved, the firm chooses its prices to maximize its revenue less downconversion costs, where the revenue function is given by Equations (1) and (2) above, but substituting for the optimum downconversion quantity using Theorem 1.

**THEOREM 2.** (a) *For any post-downconversion inventory vector  $(q_H - q_D, q_L + q_D)$  and any realization of market potential  $x$ , the optimal price vector satisfies  $a_H - p_H^* \geq a_L - p_L^*$ ; (b) *For any realization of product quantities  $(q_H, q_L)$  and market potential  $x$ , (i) no downconversion will occur, (ii) the optimal recourse price vector uniquely satisfies**

$$p_H^* = \frac{G(a_H - p_H^*)}{g(a_H - p_H^*)}, \quad p_L^* = \frac{G(a_L - p_L^*)}{g(a_L - p_L^*)}, \quad x \in \Omega_0,$$

$$p_H^* = a_H - G^{-1}\left(\frac{q_H}{x}\right), \quad p_L^* = \frac{G(a_L - p_L^*)}{g(a_L - p_L^*)}, \quad x \in \Omega_1,$$

$$p_H^* = a_H - G^{-1}\left(G(a_L - p_L^*)\sqrt{\frac{g(a_H - p_H^*)}{g(a_L - p_L^*)}}\right),$$

$$p_L^* = \frac{G(a_L - p_L^*)}{g(a_L - p_L^*)}, \quad x \in \Omega_2,$$

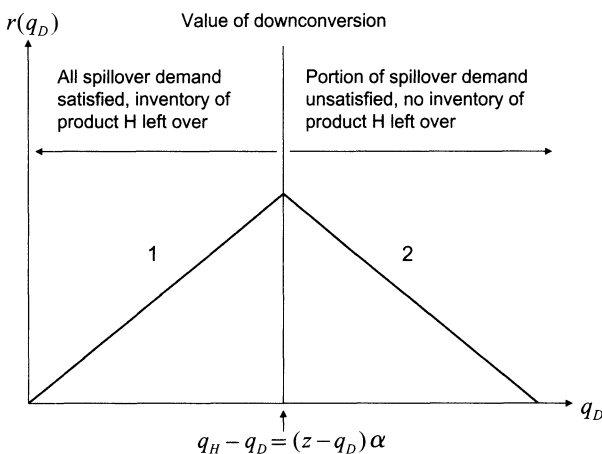
$$p_H^* = a_H - G^{-1}\left(\frac{q_H + q_L}{x}\right),$$

$$p_L^* = a_L - G^{-1}\left(\frac{q_H + q_L}{x}\right), \quad x \in \Omega_3,$$

where  $\Omega_0, \Omega_1, \Omega_2,$  and  $\Omega_3$  partition the market space, and are given by

$$\Omega_0: x \leq \frac{q_H}{(a_H - G^{-1}(q_H/x))g(G^{-1}(q_H/x))},$$

**Figure 2** Downconversion





$$\Omega_1: \frac{q_H}{(a_H - G^{-1}(q_H/x))g(G^{-1}(q_H/x))} < x$$

$$\leq \frac{q_H}{\sqrt{g(G^{-1}(q_H/x))\nu_L G(a_L - \nu_L)}},$$

$$\Omega_2: \frac{q_H}{\sqrt{g(G^{-1}(q_H/x))\nu_L G(a_L - \nu_L)}} < x$$

$$\leq \frac{q_L + q_H / \sqrt{g(a_H - \underline{p}_H)/g(a_L - \nu_L)}}{G(a_L - \nu_L)},$$

$$\Omega_3: x > \frac{q_L + q_H / \sqrt{g(a_H - \underline{p}_H)/g(a_L - \nu_L)}}{G(a_L - \nu_L)},$$

where  $\underline{p}_H = \max\{0, a_H - G^{-1}(q_H/(x - (q_L/G(a_L - \nu_L))))\}$  and  $\nu_L$  is the unique solution to  $\nu = G(a_L - \nu)/g(a_L - \nu)$ .

We note that the optimal prices are increasing in the market potential, reflecting the fact that the firm can charge higher prices when demand is high relative to supply.

**COROLLARY 1.** Under recourse pricing, if the realized market potential  $x \in \Omega_0 \cup \Omega_1$ , then it is optimal for the firm to price the two products such that customers strictly prefer H to L. Otherwise, when the market potential  $x \in \Omega_2 \cup \Omega_3$ , the firm prices the products such that the customers are indifferent.

The above corollary tells us that, regardless of the product-quantity or market-potential realizations, it is always optimal to price the products to induce a preference for H over L. As a consequence, downconversion will never occur under recourse pricing if prices are set optimally. In Online Appendix F we investigate a model in which there is residual market uncertainty when the firm makes its downconversion decision. Although we fully characterize the optimal downconversion quantity as a function of the product prices and realized production quantities, we have not been able to extend Corollary 1 to this more general model. We refer the reader to Online Appendix F for further discussion and an exploration of the model with residual uncertainty.

For the rest of this subsection, we turn our attention to the special case of the outside utility  $G(\cdot)$  being uniformly distributed. Without loss of generality, we assume  $G(\cdot) \sim U(0, 1)$  and  $0 \leq a_L \leq a_H \leq 1$ .

**COROLLARY 2.** If  $G(\cdot) \sim U(0, 1)$ , the optimal recourse price vector  $\mathbf{p}^*(q_H, q_L, x)$  is given by

$$\mathbf{p}^*(q_H, q_L, x) = \left(\frac{a_H}{2}, \frac{a_L}{2}\right), \quad \Omega_0: x \leq \frac{2q_H}{a_H},$$

$$\mathbf{p}^*(q_H, q_L, x) = \left(a_H - \frac{q_H}{x}, \frac{a_L}{2}\right), \quad \Omega_1: \frac{2q_H}{a_H} < x \leq \frac{2q_H}{a_L},$$

$$\mathbf{p}^*(q_H, q_L, x) = \left(a_H - \frac{a_L}{2}, \frac{a_L}{2}\right),$$

$$\Omega_2: \frac{2q_H}{a_L} < x \leq \frac{2(q_H + q_L)}{a_L}$$

$$\mathbf{p}^*(q_H, q_L, x) = \left(a_H - \frac{q_H + q_L}{x}, a_L - \frac{q_H + q_L}{x}\right),$$

$$\Omega_3: x > \frac{2(q_H + q_L)}{a_L}.$$

The above corollary demonstrates that the optimal prices are nondecreasing in the realized market potential and that the optimal prices always induce the customer preference for product H over L. We can use the optimal prices from Corollary 2 to develop an expression for the optimal (recourse) revenue  $\mathbf{r}_r^*(q_H, q_L, x)$  as a function of the realized quantities and market potential,

$$\mathbf{r}_r^*(q_H, q_L, x) = \left(\frac{a_H^2}{4}\right)x, \quad x \in \Omega_0,$$

$$\mathbf{r}_r^*(q_H, q_L, x) = \left(a_H - \frac{q_H}{x}\right)q_H, \quad x \in \Omega_1,$$

$$\mathbf{r}_r^*(q_H, q_L, x) = (a_H - a_L)q_H + \left(\frac{a_L^2}{4}\right)x, \quad x \in \Omega_2,$$

$$\mathbf{r}_r^*(q_H, q_L, x) = \left(a_H - \frac{q_H + q_L}{x}\right)q_H$$

$$+ \left(a_L - \frac{q_H + q_L}{x}\right)q_L, \quad x \in \Omega_3.$$

As one would expect, the optimal revenue is nondecreasing in the market potential  $x$ , the realized quantity  $q_H$ , and the customer valuations  $a_H$  and  $a_L$ . In addition, the optimal revenue is nonincreasing in the realized quantity  $q_L$  because an increase in  $q_L$  necessitates a decrease in  $q_H$ .

We are now in a position to characterize the firm's optimal production quantity  $Q^*$ . The firm chooses  $Q$  to maximize its expected profit,  $\Pi_r(Q) = -c_p Q + E_{\bar{y}, \bar{x}}[\mathbf{r}_r^*(Q, y, x)]$ , where  $\mathbf{r}_r^*(Q, y, x)$  is obtained by substituting  $q_H = yQ$  and  $q_L = (1 - y)Q$  into the above expressions for  $\mathbf{r}_r^*(q_H, q_L, x)$ . The first term is the production cost, and the second term is the expected revenue, where the expectation is taken over the yield and market potential random variables. We note that there is no cost incurred after uncertainties are resolved, because we proved above that downconversion will not occur if prices are set optimally. The expected profit function is

$$\Pi_r(Q) = \int_0^1 \left\{ \int_0^{2yQ/a_H} \frac{a_H^2}{4} x dF_X(x) \right.$$

$$\left. + \int_{2yQ/a_H}^{2yQ/a_L} \left(a_H - \frac{yQ}{x}\right)yQ dF_X(x) \right.$$

**Table 1** Numeric Study Design

	$a_H$	$a_L$	$c_P$	$c_D$	Market mean	Market spread	Yield mean	Yield spread
Mean	0.8	0.4	0.20	0	100	30	0.6	0.12
Range	Fixed	0.2, 0.6 (5)	0.16, 0.24 (5)	Fixed	Fixed	10, 50 (5)	0.40, 0.80 (5)	0.04, 0.20 (5)

Note. Number of scenarios are in parentheses.

$$\begin{aligned}
 &+ \int_{2yQ/a_L}^{2Q/a_L} \left( (a_H - a_L)yQ + \frac{a_L^2}{4}x \right) dF_X(x) \\
 &+ \int_{2Q/a_L}^{\infty} \left( \left( a_H - \frac{Q}{x} \right) yQ \right. \\
 &\left. + \left( a_L - \frac{Q}{x} \right) (1-y)Q \right) dF_X(x) \Big\} dF_Y(y) - c_P Q.
 \end{aligned}$$

Note that  $\Pi_r(Q)$  is concave in  $Q$  and so the optimal production quantity  $Q^*$  is given by the first-order condition. A closed form solution to  $Q^*$  will not, however, typically exist.

**4.3. The Value of Recourse Pricing and Operational Improvements**

We now investigate the impact of recourse pricing on the firm’s expected profit. (We note that the unabridged version of the paper also investigates the effect of recourse pricing on the optimal production quantity and finds that the effect is quite complex.) Clearly, the optimal profit under recourse pricing is always at least as large as the optimal profit under advanced pricing, because recourse pricing can always obtain the same profit as advanced pricing by simply setting the recourse price vector equal to the advance price vector for all yield and market realizations.

It is of interest to understand the relative value (i.e., the relative increase in expected profit) that can be gained by implementing recourse pricing, and how firm and market characteristics influence the relative value. To address such questions, we designed a comprehensive numeric study to investigate the influence of market uncertainty, expected yield, yield uncertainty, production cost, and customer valuation of the low-quality product.

As is common in the marketing and operations literatures, we scaled the valuations and utilities to lie between 0 and 1 in our numeric study. Therefore, costs and resulting prices are relative to these scaled utilities. We used a doubly truncated (at 0 and 1) normal distribution  $DTN(\mu = 0.5, \sigma = 1)$  for the outside-option utility, a discretized normal distribution for the market potential, and a discretized beta distribution for the yield.<sup>2</sup> We conducted a full-factorial study using the parameter values illustrated in Table 1. The

expected yield and yield-uncertainty values are representative of the semiconductor industry (see Wang et al. 2004). We chose the remaining parameters so that the weighted average profit margin at the midpoint of the parameter space is approximately 100%, a gross margin found in the semiconductor industry (see, for example, the consolidated statements on income on page 46 of Intel’s 2004 annual report).

The average percentage increase in expected profit gained by moving from advance to recourse pricing was 7.98% over all the problem instances, indicating that substantial benefit can be gained by recourse pricing. We now present our findings on the influence of market and firm characteristics on the relative value of recourse pricing over advanced pricing. When reporting the relative increase in expected profit for a particular parameter value (or combination of parameter values) in the tables that follow, we present the average percentage increase across all instances that have that particular parameter value (or combination of parameter values).

We found that the value of recourse pricing was typically (but not always) increasing in market uncertainty and in yield uncertainty. The reason for this is that recourse pricing enables the firm to better manage uncertainty, and so recourse pricing is more advantageous if uncertainty increases. The value of recourse pricing was typically increasing in the production cost, but we have not been able to identify an intuitive explanation for this effect. Whereas the value of recourse pricing was increasing in the expected yield for a uniformly distributed yield,<sup>3</sup> there was

note that a uniform (0, 1) distribution is often assumed for utilities in the marketing and operations literatures. We conducted a second study in which we used uniform distributions for the utility, market potential and yield. The magnitude and directional results of this study were very similar, with the exception of the directional effect of the expected yield, discussed later. As such, we do not report the numbers from that second study but they are available upon request.

<sup>3</sup> The reason that recourse pricing becomes more advantageous as expected yield increases (in the uniform distribution case) is linked to the fact that there is relatively more inventory of H than L as expected yield increases. This gives the firm more latitude in pricing product H but less in pricing product L. The increased latitude in the pricing of H appears to be especially beneficial for recourse pricing. Note that the expected profits under both recourse and advanced pricing increase in the expected yield and so does the magnitude of their difference. In addition, the relative value of recourse pricing also increases in the expected yield.

<sup>2</sup> Bitran and Leong (1992) and Bitran and Gilbert (1994) also used beta distributions for the yield. Other authors, e.g., Wang et al. (2004), have used a uniform distribution to represent the yield. We

no clear directional effect for the case of a beta-distributed yield. We conjecture that the lack of a clear directional effect is due to the fact that the shape of the beta distribution changes as the mean changes. Tables reporting the relative change in expected profit (due to recourse pricing) as a function of market uncertainty, yield uncertainty, expected yield, and production cost can be found in the unabridged version of the paper.

We now consider the influence that the customer valuation of product L,  $a_L$ , has on the value of recourse pricing. We first note that an increase in  $a_L$  mitigates the impact of yield uncertainty as a high realization of the “byproduct” L is less of an issue if that byproduct is highly valued. In fact, in the extreme case where  $a_H = a_L$ , then yield uncertainty is irrelevant because production results in a random split of equally valued products; in essence, the system becomes a perfect-supply, single-product system. If this mitigation effect was the only effect of increasing  $a_L$ , then the value of recourse pricing would be decreasing in  $a_L$ . There is, however, another effect of increasing  $a_L$ ; the firm’s inventory is more valuable. The firm will always charge a price for product L that is lower than  $a_L$ . Increasing  $a_L$  increases the feasible price region, and this increase enables recourse pricing to be more effective and, thus, more valuable. We then have two counterbalancing effects: the uncertainty-mitigation effect and the pricing-latitude effect. Table 2 presents the relative value of recourse pricing as a function of  $a_L$ . We see that for low  $a_L$ , the mitigation effect dominates and the value of recourse pricing is decreasing in  $a_L$ . As  $a_L$  increases further, uncertainty becomes less significant due to mitigation, and so the pricing-latitude effect starts to dominate with the result that value of recourse pricing starts to increase in  $a_L$ .

Implementing recourse pricing is not the only option for improving the expected profit of a coproduction system. The firm might instead prefer to invest in some operational improvement effort. Operational improvements include reducing the production cost, increasing the expected yield of the high-quality product H, reducing yield uncertainty, or increasing the valuation of the low-quality product L. This last option might reflect an actual increase in the quality of L, or alternatively a marketing effort to increase the customer’s valuation of the original quality of L. To compare the value of recourse pricing to operational improvement, we calculated, for every problem instance, the necessary improvement in a given dimension (e.g. production-cost reduction) to deliver an equivalent benefit to recourse pricing. On average, a 8.7% reduction in production cost, or a 6.9% increases in expected yield, or a 37.9% increase in low product valuation was needed to deliver equivalent

**Table 2** Relative Value of Recourse Pricing as a Function of  $a_L$  (in %)

Market spread	$a_L$				
	0.2	0.3	0.4	0.5	0.6
30	15.18	9.13	5.77	5.24	5.81
40	16.35	10.18	7.17	6.71	7.12
50	16.49	10.46	7.97	7.84	8.28
Average	16.01	9.92	6.97	6.60	7.07

benefit to recourse pricing. We note that in 21.02%<sup>4</sup> of the cases, the maximum possible increase in expected yield did not deliver as large a benefit as did recourse pricing. Completely eliminating yield uncertainty did not deliver as large a benefit as did recourse pricing in 97.3% of the instances. In these cases, the benefit of completely eliminating yield uncertainty delivered an average of 89% of the value of recourse pricing. Such numbers indicate that implementing recourse pricing delivers approximately the same benefit as eliminating yield uncertainty, suggesting that recourse pricing is very effective at managing yield uncertainty. We note that as market uncertainty increases, the operational improvements needed to match recourse pricing also increased. This is because recourse pricing is particularly advantageous when market uncertainty is high. The numbers presented here relate only to the value of operational improvements and recourse pricing, and do not account for the relative difficulty or cost of achieving such improvements or implementing recourse pricing. The preference a firm has for a particular operational improvement effort or for recourse pricing will therefore depend on the various implementation costs.

## 5. A Two-Class Coproduction Model with Pricing

In this section, we consider the two-class coproduction model (labeled CPP2). Recall that without loss of generality we index the classes  $i = 1, 2$  such that  $a_{1H} \geq a_{2H}$ .

### 5.1. Allocation Policy

Unlike in the single-class case, the firm now faces an allocation decision, that is, how to ration its inventory among the two classes in the event of a shortage. A natural allocation policy to consider is a priority-based allocation policy, whereby the firm selects one class as being the priority class, and members of that class make their purchasing decisions before members of the nonpriority class. In what follows, let  $K \in \{H, L\}$  and let  $\bar{K}$  denote the complement. Recall that  $G_i(a_{iK} - p_K)$ ,  $i = 1, 2$ ;  $K \in \{H, L\}$ , is the fraction

<sup>4</sup> The equivalent number is 0.01% for the study with a uniform yield distribution.

of class  $i$  customers willing to buy product  $K$ . For  $a_{iK} - p_K \geq a_{i\bar{K}} - p_{\bar{K}}$  (i.e., class  $i$  prefers  $K$  to  $\bar{K}$ ), denote the spillover ratio  $G_i(a_{i\bar{K}} - p_{\bar{K}})/G_i(a_{iK} - p_K)$  as  $s_{i\bar{K}}$ . In other words,  $s_{i\bar{K}}$  is the fraction of class  $i$  customers willing to spill over to product  $\bar{K}$  if their first choice  $K$  runs out.

**THEOREM 3.** (a) For a given price vector  $\mathbf{p}$ , (i) the firm is indifferent between all allocation policies if class 1 and 2 have different product preferences. (ii) If the firm uses a prioritization policy and if both classes prefer  $K$  to  $\bar{K}$ , then priority should be given to class 1 if  $s_{1\bar{K}} < s_{2\bar{K}}$  and class 2 if  $s_{1\bar{K}} > s_{2\bar{K}}$ . The firm is indifferent if  $s_{1\bar{K}} = s_{2\bar{K}}$ . (b) (i) A priority-based allocation rule is optimal (among all allocation policies) for recourse allocation regardless of the timing, advance or recourse, of the pricing decision. (ii) A priority-based allocation rule is optimal (among all allocation policies) for advanced allocation under advanced pricing.

From the above theorem, we see that the firm is indifferent between allocation policies if the two classes prefer different products. As mentioned in the introduction, the existing coproduction literature not only assumed exogenous prices but implicitly assumed that different customer classes preferred different products. This is one of the reasons that the allocation question did not arise in such papers, that, and the fact that spill overs were not allowed.

The theorem also proves that if both classes prefer the same product, say  $K \in \{H, L\}$ , then the firm should prioritize the class with the lower spillover ratio. Why is this? If there is insufficient inventory of  $K$ , and so the firm has to ration  $K$ , the firm’s revenue is increasing in the number of unsatisfied customers preferring to purchase the other product rather than taking their outside option. Thus, priority is based on the spillover ratios.

As mentioned in the introduction, we consider both advanced allocation in which the firm chooses its allocation policy before yield and market uncertainties are resolved and recourse allocation in which the firm postpones its choice of allocation policy until after uncertainties are resolved. Clearly recourse allocation can be no worse than advanced allocation, because the firm could simply choose the same allocation policy as under advanced allocation for all realizations of yield and market potentials. Theorem 3 established that a priority policy is optimal under recourse pricing and under advanced pricing if advanced allocation is done. We note that, although we have not proven that a priority allocation policy is optimal for advanced allocation under recourse pricing, we restrict attention to a prioritization policy (unless otherwise stated), and therefore use the term prioritization rather than allocation in what follows.

Theorem 3 established that the optimal priority depends only on the spillover ratios. The spillover

ratios depend only on the price vector  $\mathbf{p}$  and valuations  $a_{iK}$ ,  $i = 1, 2$ ;  $K \in \{H, L\}$ . The valuations are not uncertain. However, in recourse pricing, the optimal  $\mathbf{p}$  depends on market and yield realizations, and therefore the optimal priority class cannot be determined a priori. Therefore, recourse prioritization is preferred to advanced prioritization under recourse pricing. In advanced pricing, there is no uncertainty about the price vector  $\mathbf{p}$  and therefore the optimal priority class can be determined a priori. The following result then follows immediately.

**REMARK.** Recourse prioritization offers no value if the firm makes its pricing decision in advance.

We note that implementation of a prioritization policy assumes that the firm knows a customer’s class. This is a reasonable assumption in many coproduction systems because the firm will often know how a customer uses its product and will therefore be able to infer the customer’s valuation type. However, if the firm is not able to distinguish between customer classes, then randomized allocation is the most likely policy. We refer the reader to Online Appendix D for an analytical and numeric treatment of randomized allocation. Our study found that the value of knowing a customer type, i.e., the ability to implement prioritization, can sometimes be significant (see Online Appendix D).

We now proceed to develop the firm’s revenue expression given it uses a prioritization policy. We use  $j$  to denote the priority class and  $\bar{j}$  to denote the nonpriority class. For notational clarity in what follows, define  $d_{iK} = x_i G_i(a_{iK} - p_K)$ ,  $i = 1, 2$ ;  $K \in \{H, L\}$ , that is,  $d_{iK}$  is the quantity of class  $i$  customers who prefer  $K$ , at price  $p_K$ , to their outside option. For a price vector  $\mathbf{p} = (p_H, p_L)$ , realized quantities  $\mathbf{q} = (q_H, q_L)$ , and market-potential realizations  $\mathbf{x} = (x_1, x_2)$ , the firm’s revenue as a function of the downconversion quantity  $q_D$  is

$$r(q_D) = p_H \min \left\{ \sum_{i=1}^2 d_{iH}, (q_H - q_D) \right\} + p_L \min \{ (d_{jH} - (q_H - q_D))^+ s_{jL}, (q_L + q_D) \}, \quad \mathbf{p} \in \Gamma_1,$$

$$r(q_D) = p_H \min \{ d_{jH} + (d_{\bar{j}L} - ((q_L + q_D) - (d_{jH} - (q_H - q_D))^+ s_{jL}))^+ s_{\bar{j}H}, (q_H - q_D) \} + p_L \min \{ d_{\bar{j}L} + (d_{jH} - (q_H - q_D))^+ s_{jL}, (q_L + q_D) \}, \quad \mathbf{p} \in \Gamma_2,$$

$$r(q_D) = p_H \min \{ d_{jH} + (d_{\bar{j}L} - (q_L + q_D))^+ s_{\bar{j}H}, (q_H - q_D) \} + p_L \min \{ d_{\bar{j}L} + (d_{jH} - ((q_H - q_D) - (d_{\bar{j}L} - (q_L + q_D))^+ s_{\bar{j}H}))^+ s_{jL}, (q_L + q_D) \}, \quad \mathbf{p} \in \Gamma_3,$$

$$r(q_D) = p_H \min\{(d_{jL} - (q_L + q_D))^+ s_{jH} + (d_{\bar{j}L} - ((q_L + q_D) - d_{jL}))^+ s_{\bar{j}H}, (q_H - q_D)\} + p_L \min\left\{\sum_{i=1}^2 d_{iL}, (q_L + q_D)\right\}, \quad \mathbf{p} \in \Gamma_4,$$

where  $\Gamma_1, \Gamma_2, \Gamma_3,$  and  $\Gamma_4$  partition the pricing space and are given by

$$\begin{aligned} \Gamma_1: & p_H - p_L \leq \min_i \{a_{iH} - a_{iL}\}, \\ \Gamma_2: & a_{\bar{j}H} - a_{\bar{j}L} < p_H - p_L \leq a_{jH} - a_{jL}, \\ \Gamma_3: & a_{jH} - a_{jL} < p_H - p_L \leq a_{\bar{j}H} - a_{\bar{j}L}, \\ \Gamma_4: & p_H - p_L > \max_i \{a_{iH} - a_{iL}\}. \end{aligned}$$

We note that  $\Gamma_2$  and  $\Gamma_3$  cannot exist simultaneously, that is, depending on the values of  $a_{iK}, i = 1, 2; K \in \{H, L\}$ , either  $\Gamma_2$  or  $\Gamma_3$  will exist. We note that the revenue expression under randomized allocation can be found in Online Appendix D.

### 5.2. Downconversion

Using the above revenue expression, we can now derive the firm's optimal downconversion quantity  $q_D$  under a priority allocation policy. Recall that the downconversion cost is  $c_D$  and that  $j$  denotes the priority class and  $\bar{j}$  the nonpriority class.

**THEOREM 4.** For a price vector  $\mathbf{p} = (p_H, p_L)$ , realized quantities  $\mathbf{q} = (q_H, q_L)$ , and market-potential realizations  $\mathbf{x} = (x_1, x_2)$ , the optimal downconversion quantity  $q_D^* = 0$  if (a) both class of customers prefer product H to L; or (b) class  $j$  customers prefer product L and  $c_D \geq p_L - p_H s_{jH}$ ; or (c) class  $j$  customers prefer product H and class  $\bar{j}$  customers prefer product L and  $c_D \geq p_L - p_H s_{\bar{j}H}$ . Otherwise, the optimal downconversion quantity is given by  $q_D^* = \min\{z, \hat{q}_D\}$ , where

$$\begin{aligned} \hat{q}_D &= \frac{(q_H - d_{jH}) - (d_{\bar{j}L} - q_L) s_{\bar{j}H}}{1 - s_{\bar{j}H}}, \\ & \mathbf{p} \in \Gamma_2, d_{jH} < q_H \cap d_{\bar{j}L} > q_L \cap (d_{\bar{j}L} - q_L) s_{\bar{j}H} < q_H - d_{jH}, \\ \hat{q}_D &= \frac{(q_H - d_{\bar{j}H}) - (d_{jL} - q_L) s_{jH}}{1 - s_{jH}}, \\ & \mathbf{p} \in \Gamma_3, d_{\bar{j}H} < q_H \cap d_{jL} > q_L \cap (d_{jL} - q_L) s_{jH} < q_H - d_{\bar{j}H}, \\ \hat{q}_D &= \frac{q_H - (\sum_{i=1}^2 d_{iL} - q_L) s_{\bar{j}H}}{1 - s_{\bar{j}H}}, \\ & \mathbf{p} \in \Gamma_4, d_{jL} \leq q_L \cap \sum_{i=1}^2 d_{iL} > q_L \cap \left(\sum_{i=1}^2 d_{iL} - q_L\right) s_{\bar{j}H} < q_H, \\ \hat{q}_D &= \min\{z_1, \hat{q}_{D1}\} + \min\{z_2, \hat{q}_{D2}\}, \\ & \mathbf{p} \in \Gamma_4, d_{jL} > q_L \cap (d_{jL} - q_L) s_{jH} + d_{\bar{j}L} s_{\bar{j}H} < q_H, \\ \hat{q}_D &= 0, \quad \text{otherwise,} \end{aligned}$$

where

$$\begin{aligned} \hat{q}_{D1} &= \frac{q_H - (d_{jL} - q_L) s_{jH} - d_{\bar{j}L} s_{\bar{j}H}}{1 - s_{jH}}, \\ \hat{q}_{D2} &= \frac{(q_H - (d_{jL} - q_L - \hat{q}_{D1}) s_{jH} - \hat{q}_{D1} - d_{\bar{j}L} s_{\bar{j}H})^+}{1 - s_{\bar{j}H}}, \end{aligned}$$

and

$$\begin{aligned} z &= \max\{0, d_{\bar{j}L} - q_L\}, \quad \mathbf{p} \in \Gamma_2, \\ z &= \max\{0, d_{jL} - q_L\}, \quad \mathbf{p} \in \Gamma_3, \\ z &= \max\left\{0, \sum_{i=1}^2 d_{iL} - q_L\right\}, \quad \mathbf{p} \in \Gamma_4, \\ z_1 &= \max\{0, d_{jL} - q_L\}, \quad z_2 = \max\{0, d_{\bar{j}L}\}, \quad \mathbf{p} \in \Gamma_4. \end{aligned}$$

An implication of this theorem is that the firm will not downconvert unless there is at least one customer class that prefers product L to product H, and even then will only downconvert if the cost is not too high. An equivalent theorem for randomized allocation can be found in Online Appendix D.

### 5.3. Pricing

In §4 we proved that in the single-class case, the firm prices the products such that H is preferred to L and that, therefore, downconversion does not occur. The following theorem establishes that in the two-class case, the firm may price the products such that one class prefers product L and that downconversion may occur.

**THEOREM 5.** The optimal price vector  $\mathbf{p}^* = (p_H^*, p_L^*)$  may induce one class to prefer L to H, that is  $a_{iL} - p_L^* > a_{iH} - p_H^*$  for some  $i = 1, 2$ . Furthermore, a strictly positive downconversion quantity may be optimal.

A fundamentally different result therefore holds when we move from the single-class case to the two-class case. The firm's inability to price discriminate lies at the heart of this result. The firm is constrained to offer the same price vector to both classes and this constraint can result in the best single price vector inducing one class to prefer L to H. In this case, downconversion can be optimal so long as the downconversion cost is not too high.

For the single-class model (CPP1), we were able to obtain implicit solutions for the optimal recourse price vector (and closed form solutions in the case of a uniform utility distribution). Unfortunately the two-class model does not in general lend itself to solving for the optimal recourse prices. However, in the special case where  $a_{iL} = 0, i = 1, 2$ , we are able to solve for the optimal price. This special case (RYP2) is a random-yield, single-product model, and a full analysis of RYP2 can be found in Online Appendix B. As

mentioned, we have not developed implicit expressions for the optimal recourse prices for the general CPP2 model. However, the optimal prices (recourse and advanced) can be readily found numerically.

#### 5.4. The Value of Recourse Actions

We carried out an extensive numeric study to investigate the value of recourse pricing, downconversion and recourse prioritization. We assumed a doubly truncated (at 0 and 1) normal  $DTN(0.5, 1)$  distributions for the utilities of outside options.<sup>5</sup> Yield uncertainty was represented by a discretized beta distribution, where the yield standard deviation was varied from 0.04 to 0.20 in increments of 0.04. The expected yield of product H was varied from 0.4 to 0.8 in increments of 0.1. The expected market potential  $\mu_{X_1}$  and product valuations ( $a_{1H}, a_{1L}$ ) for class 1 customers were fixed at  $\mu_{X_1} = 100$  and  $(a_{1H}, a_{1L}) = (0.8, 0.4)$ . The expected market potential for class 2 customers  $\mu_{X_2}$  was varied from 40 to 160 in increments of 30. Market uncertainty was represented by three scenarios ( $\mu_{X_i} - s_{X_i}, \mu_{X_i}, \mu_{X_i} + s_{X_i}$ ) with associated probabilities (0.25, 0.5, 0.25), where  $s_{X_i}$  is defined as the spread of the distribution. The scenarios were set up such that the two market potentials were independent. The market spread for each class was simultaneously varied from 10 to 50<sup>6</sup> in increments of 10, i.e., the markets have equal variance in each instance. Class 2's valuation of product H was varied from 0.3 to 0.6 in increments of 0.1. Class 2's valuation of product L was varied from 0.2 to  $a_{2H} - 0.1$  in increments of 0.1. The production cost was fixed at 0.20. Downconversion was either impossible (i.e., infinite cost) or possible at a marginal cost of 0.5% of the production cost.<sup>7</sup> The total number of problem instances was thus 12,000. We note that the numbers reported below for advanced prioritization (i.e., a fixed priority is chosen before uncertainties are resolved) reflect the optimal advanced priority choice.

In a separate study, we investigated the effect of market correlation by creating instances with correlation coefficients of  $-1, 0,$  and  $1$ . We fixed the expected yield at 0.6, the yield spread at 0.12 and the production cost at 0.2. All other parameters were varied as described above. This study had 1,440 instances.

<sup>5</sup> As with the single-class study, we also carried out a separate investigation when utilities, market potentials and the yield all had uniform distributions. Again, the results were very similar, unless otherwise stated in the paper, and are not reported here. They are available upon request.

<sup>6</sup> We note that in the case of  $\mu_{X_2} = 40$ , the market spread for class 2 was not increased beyond 40 as this is the maximum spread for a distribution centered at 40.

<sup>7</sup> We also investigated the downconversion cost at 5% of the production cost. The value of downconversion (reported later) was slightly lower, but the other results were similar to those for the 0.5% case.

**Table 3** Relative Value of Downconversion as a Function of Market Uncertainty (in %)

	Market spread				
	10	20	30	40	50
Advance pricing					
Average	0.96	0.96	0.99	1.09	1.39
Maximum	10.15	9.96	10.75	12.18	12.90
Recourse pricing					
Average	0.11	0.11	0.12	0.13	0.15
Maximum	4.10	3.89	3.42	2.93	2.67

For advanced (recourse) prioritization, the average value of recourse pricing, i.e., the increase in expected profit over advanced pricing, was 10.02% (10.03%) when downconversion was possible and 11.06% (11.07%) when downconversion was not possible. The influence of production and market characteristics on the value of recourse pricing were similar to the single-class case and so we do not discuss them again here.

For advanced prioritization, the average value of downconversion, i.e., the increase in expected profit over no downconversion, was 1.07% for advanced pricing and 0.12% for recourse pricing. Such numbers might suggest that downconversion offers little value. The average value, however, obscures the fact that downconversion can be very valuable in certain instances. The maximum value of downconversion was 12.90% for the advanced pricing case.<sup>8</sup> The value of downconversion is influenced primarily by two factors. One factor is the probability of mismatches in supply and demand for product L; the more likely such mismatches are, the more likely the firm will need to engage in downconversion. The other factor is the constraint on how much the firm can convert, i.e., the less of product H it has, the less it can convert. Tables 3, 4, and 5 present the average and maximum values of downconversion as market uncertainty (as represented by the spread), yield uncertainty, and expected yield increase. We see that the value of downconversion is increasing in market uncertainty, yield uncertainty, and expected yield. The value is increasing in market uncertainty and yield uncertainty because supply–demand mismatches are more likely as uncertainty grows. The value is increasing in expected yield because more product H is produced as the expected yield increases. Tables 3, 4, and 5 also show that the value of downconversion is significantly decreased if recourse pricing is used. This finding, in conjunction with the

<sup>8</sup> In the study with uniformly distributed utilities, market potential, and yield, the maximum value was 25.32%, but the average values were essentially the same as those reported above, i.e., 1.03% compared to 1.07%, and 0.13% compared to 0.12%.

**Table 4** Relative Value of Downconversion as a Function of Yield Uncertainty (in %)

	Yield spread				
	0.04	0.08	0.12	0.16	0.20
Advance pricing					
Average	0.44	0.68	1.03	1.40	1.78
Maximum	9.90	9.89	10.17	10.85	12.90
Recourse pricing					
Average	0.06	0.08	0.11	0.16	0.21
Maximum	1.76	2.07	2.67	3.62	4.10

fact that recourse pricing provides a higher average value (11.07%) than does downconversion (1.07%), suggests recourse demand management, i.e., pricing, is more valuable than recourse supply management, i.e., downconversion.

Our investigation of market correlation found the value of downconversion to be decreasing in market correlation. This makes sense as one of the necessary conditions for downconversion to occur is that there be an excess of one product and a shortage of the other. The more positively correlated the markets the less likely are such situations.

We now turn our attention to the value of recourse prioritization, i.e., the value of postponing the class prioritization decision until uncertainties are resolved. Recourse prioritization allows the firm to postpone the prioritization decision until after uncertainties are resolved. We already have proven that recourse prioritization offers no value under advanced pricing, but that it can add value under recourse pricing (because the priority depends on the prices and the valuations, and the optimal prices cannot be known in advance). However, the other determinant, i.e., the product valuations, is not uncertain, and so one might conjecture that recourse prioritization might be of little value. For recourse pricing, the average (max) value of recourse prioritization, i.e., the increase in expected profit over advanced prioritization, was 0.01% (0.41%) in our numeric study. This number suggests that the value of recourse prioritization is negligible on average, even for high market uncertainties. Customer

**Table 5** Relative Value of Downconversion as a Function of Expected Yield (in %)

	Expected yield				
	0.4	0.5	0.6	0.7	0.8
Advance pricing					
Average	0.48	0.63	0.97	1.51	1.74
Maximum	5.76	7.65	11.82	11.38	12.90
Recourse pricing					
Average	0.01	0.04	0.07	0.16	0.32
Maximum	0.30	0.58	1.10	2.22	4.10

valuations appear to be a more critical driver of the prioritization decision, and the fact that valuations are certain means that recourse prioritization is of little value.

## 6. Conclusions

Coproduction systems are prevalent in many industries. Such systems present many challenges; the firm must make pricing, quantity, downconversion and allocation decisions in an environment of potentially high uncertainties in both supply and demand. Previous literature has focused on the quantity and downconversion decision assuming exogenous prices, differing customer-class preferences, and costless downconversion. We jointly consider the pricing, quantity, downconversion, and allocation problem for a single-period, two-product problem, using a utility-maximizing demand model to ensure a customer's behavior is completely consistent with her product valuations and the announced prices. Furthermore, we allow for costly downconversion. We consider both advanced and recourse pricing and advanced and recourse allocation.

We establish that downconversion will never occur in the single-customer class case if prices are set optimally. (The unabridged version of the paper considers an alternative model in which customers are willing to wait for downconversion. We prove that downconversion can occur even if prices are set optimally in that case.) We explicitly characterize the optimal recourse prices and show recourse pricing is very beneficial, delivering the same value as a 9% reduction in production cost or approximately the same value as completely eliminating supply uncertainty. With two customer classes, the firm must decide on an allocation policy. We prove that a priority allocation policy is optimal for recourse allocation. In addition, we show that downconversion can be optimal when there are two customer classes. Although the average value of downconversion is not very high, it can be very valuable in environments of high demand and supply uncertainty. Recourse prioritization was found to add little value, primarily because the valuation heavily influences the choice of the priority class and valuations are not uncertain. Recourse pricing was found to be very beneficial, suggesting that recourse demand management is more valuable than recourse supply management. It may, however, be harder to implement recourse pricing.

As with the existing stochastic-demand coproduction literature, we have limited our attention to the single-period problem. An extension to the multi-period setting (even for the exogenous price case) for stochastic-demand problem would be of great interest, but would also be very challenging. Even



the deterministic-demand papers resort to heuristics for the multiperiod problem. We conclude by noting that coproduction systems, although complex from an analytical nature, are rich in opportunities for research in risk management and product-variety management. The high degree of supply and demand uncertainty make coproduction systems attractive candidates for those interested in the financial and operation hedging of risk. All coproduction research to date has assumed risk neutrality. The multiproduct nature of coproduction systems should make them attractive to those interested in product variety or assortment problems. For example, in many coproduction systems the output varies continuously over the quality dimension. A firm then has to decide how to segment the output into products; it might offer a high number of very tightly specified products or a small number of more loosely specified products. All research to date has assumed the product offering decision is exogenous. We hope that future research will address these and other questions.

## 7. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://mansci.journal.informs.org/>.

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